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THE UNIVERSITY OF ALBERTA

ANALYSIS OF THE WIND-SPEED PROFILES IN THE SURFACE BOUNDARY LAYER

by

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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Analysis of the Wind-Speed Profiles in the Surface Boundary Layer", submitted by Oscar Koren in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

Analyses are performed on 170 wind profiles which were obtained at the Suffield Experimental Station during the 1959-1967 period. In addition, 34 wind profiles obtained at Kerang, Australia, are considered in the analyses. The data consisted of measurements of wind speed and temperature at a number of levels up to 96 meters. A comparison of the results obtained through the use of the power law, logarithmic law and log-linear law, is made. It is found that the log-linear law represents the data in the near-adiabatic conditions more accurately than does either the logarithmic or the power law. However, under strong inversions the power law gives a better representation.

It is shown that the wind profile data used in the analyses are represented well by a modified log-linear law in the form

$$U = \frac{u_*}{k} \left[\ln \frac{z}{z_o} + \left(\frac{1}{L_o}\right)^{\frac{1}{2}} \frac{z}{\sqrt{L}} \right]$$

This modified wind profile law is compared with the Monin-Obukhov log-linear law and Brooks log-power law. It is found that the differences among these three laws are not statistically significant.

A number of results obtained in this study are found to be in agreement with those found by an independent investigation at the Suffield Experimental Station. The results from this investigation are also compared with those obtained at the Brookhaven National Laboratory and at a number of other locations.

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CHAPTER I

OUTLINE OF THE PROBLEM

1.1 Introduction

The air motion in the free atmosphere is very nearly horizontal and predominantly determined by a quasi-balance between the horizontal pressure force and the horizontal Coriolis force. This quasi-horizontal air motion is modified considerably in the surface boundary layer which extends from the earth's surface to a height of approximately 100 meters. In this layer the wind is subjected to a considerable frictional force whose magnitude varies primarily with the nature of the surface and with the temperature distribution in the vertical. Moreover, the air motion in this layer is, normally, turbulent which causes a continuous diffusion of properties from one region to another.

With the development of the theory of atmospheric turbulence at the beginning of the twentieth century it became necessary to consider the vertical structure of wind and temperature near the ground. Investigations showed that turbulence was related to a set of external parameters which had to be determined from the vertical wind profiles measured under various stability conditions. Furthermore, information on the variation of wind with height was required in many practical problems. Construction of high structures, design of airplanes and launching of satellites are only a few examples of where the wind profile information was utilized. With the rapid technological advancements in the past few years, accurate determinations of the wind profiles near the earth's surface became essential. In many applications

the variation of wind speed with height has to be expressed mathematically. No definitive law describing the variation of wind with height in the surface boundary layer exists as yet. However, several wind laws, describing the wind variation with height under various stability conditions, have been proposed. It is the aim of this thesis to investigate how well experimentally measured wind profiles can be expressed by the existing wind laws, particularly those which are most commonly used in theory and practice.

1.2 The Problem

Investigations by Frost (1948), Panofsky (1963), Hansen (1966) and others have shown that various theoretically and empirically derived equations, describing the variations of wind with height, fit the actual observations with various degrees of accuracy. The three main approaches taken to find a general wind profile equation were the logarithmic law approach, the power law approach and more recently the log-linear law approach based on the Monin-Obukhov similarity theory. Several sets of wind profile data have been obtained and analysed in the past. Most of the early data examined a relatively shallow layer of the atmosphere, generally no higher than a few meters. In most cases the early data were fitted to either the power or logarithmic law. More recently, with the construction of meteorological towers, wind profile measurements have been extended to a height of 100 meters and in a few instances even higher. During the last few years profile laws based on the similarity theory

have gained increasingly more acceptance. Panofsky and Prasad (1965) write:

> "The Monin-Obukhov similarity theory has made it possible to treat interaction of mechanical turbulence and heat convection in a meaningful manner. Thus, it is now generally accepted in the description of wind profiles near the ground".

In the analyses presented in this study the wind profile data extending from surface to heights up to 96 meters were used to investigate which of the wind profile laws represented the actual observations most accurately, and under what stability conditions. Calculations were also made to determine the parameters which appear in different wind profile laws. A statistical approach was used to establish which law was best for a particular situation. The sample of more than 200 wind profiles was considered large enough to warrant such an approach. It was hoped that this investigation of the variation of wind speed with height would add to the understanding of the atmospheric processes in the lowest layers of the atmosphere.

1.3 Instrumentation and Data

Various diffusion experiments conducted by the Department of National Defense at the Suffield Experimental Station, Alberta, over the last two decades required precise measurements of the wind profiles. The chief source of the wind profile data was the 96-m micrometeorological tower located at the station. Micrometeorological sensors were located at different heights on the tower.

The wind speed measurements were made with Sheppard-Casella 3-cup contact anemometers connected electrically to recording counters which were photographed at one-minute intervals. The sampling periods varied depending on the experimental requirements; however, the most frequent sampling period was ten minutes. The anemometers were placed at heights of 2, 8, 16, 32, 48, 64, 80, 92 and occasionally at 96 m above ground level. The temperatures were measured at heights of 0.5, 1, 4, 92 and occasionally at 13 m. For a number of wind profiles, instead of the temperature measurement at one-meter above ground, the temperature measurement at two-meters above ground was obtained. Thus, the temperature gradients for layers 4-0.5, 13-1 or 13-2 and 92-1 or 92-2 m were obtained.

Besides the 96-m tower two movable 16-m towers, with anemometers placed at 0.5, 1, 2, 4, 8 and 16 m, were used to measure the variation of wind with height during some of the diffusion experiments. Temperatures on the 16-m towers were measured at 0.5, 4 and occasionally at 8 m above ground.

The site of the 96-m tower was gently rolling prairie grass with no large obstructions within two miles. The 16-m towers were, generally, placed a few hundred feet from the 96-m tower. During the summer months the surface in the vicinity of the towers consisted of sparse grass six to ten centimeters high with very sparse stalks up to 30 cm. During the winter months the shorter grass was covered with snow and very sparse stalks up to 20 cm high protruded through the moderately smooth snow surface.

During the 1959-1967 period 170 wind profiles, together with the corresponding temperature gradients, were obtained. The wind profile measurements were taken at different times during the year and under various stability conditions. A summary of the main features of the Suffield data is given in Table 1.1. Occasional malfunctions of the instruments caused some omissions in the data; however, the number of the missing observations was small in comparison to the total number of observations.

The temperature gradients (ΔT), which were represented as the temperature differences, in degrees Fahrenheit, between the top and the bottom of the various layers, varied from -7.0 to 3.8 in the 0.5-4 m layer; from -1.0 to 7.2 in the 1-13 m layer; and from -4.6 to 24.5 in the 1-92 m layer. In order that the stability conditions in the three individual layers could be compared, the temperature difference corresponding to the adiabatic lapse rate was calculated for each layer. It was found that the temperature difference of -0.1, -0.2 and -1.6 F would produce approximately adiabatic lapse rate in the 3.5, 12 and 91 m thick layers respectively. The following stability conditions were then defined: inversion, isothermal, stable, adiabatic (neutral) and super-adiabatic. The limits for these conditions together with the number of observations in each interval are given in Table 1.2.

In addition to the Suffield data a sample of 34 wind profiles obtained at Kerang, Australia by Swinbank (1964) was analysed. The Kerang data were selected primarily as a control sample because,

Table 1.1

Summary of the main features of the Suffield Data

Profile Characteristic	Interval	Number of Obs.
	Jan - Feb - Mar	50
Time of the Year (Month)	Apr - May - Jun	5
	Jul - Aug - Sep	9
	Oct - Nov - Dec	106
	0900 - 1300	34
Time of Day (MST)	1301 - 1700	60
	1701 - 2100	66
	2101 - 2400	10
	001 - 090	4
Wind Direction (Deg. True)	091 - 180	19
	181 - 270	122
	271 - 360	20
	Missing	5
	$h \geq 80$	116
Height of Profile (m)	$80 > h \geq 50$	8
	$50 > h > 16$	5
	$h \leq 16$	41
	$t < 10$	17
Sampling Period (Min)	$t = 10$	101
	$30 \geq t > 10$	15
	$t > 30$	37
	$T < 32$	50
Surface Temperature (F)	$50 \geq T \geq 32$	35
	$70 \geq T > 50$	61
	$T > 70$	16
	Missing	8

according to the literature, it is one of the most accurate sets of observations available at the present time. The heights of the anemometers at Kerang were identical to those used in the 0.5 - 16 m layer at Suffield. Thus a direct comparison of the results between the two locations was possible. Unfortunately, the comparison was possible only for the unstable cases because the Kerang data were limited to unstable conditions.

Table 1.2

Definitions of stability conditions and number of observations in each interval (Suffield data)

Stability condition	$T_4 - T_{0.5}$		$T_{13} - T_1$ or $T_{13} - T_2$		$T_{92} - T_1$ or $T_{92} - T_2$	
	Limits (F)	No. of Obs.*	Limits (F)	No. of Obs.	Limits (F)	No. of Obs.**
Inversion	>0.0	19	>0.0	36	>0.2	64
Isothermal	0.0	2	0.0 to 0.1	8	-0.2 to 0.2	7
Stable	-	-	-0.1	2	-1.3 to -0.3	7
Neutral	-0.3 to -0.1	3	-0.4 to -0.2	3	-1.8 to -1.4	6
Super-adiabatic	<-0.3	17	<-0.4	14	<-1.8	11

* Profiles extending to a height $h \leqslant 16$ m
** Profiles extending to a height $h \geqslant 80$ m

CHAPTER II

TURBULENCE AND STABILITY PARAMETERS

2.1 The Reynolds Number

Until the beginning of the nineteenth century little was known about the laws of internal friction of fluids. Systematic investigations of the flow phenomena in pipes and channels began with Osborne Reynolds in the middle of the nineteenth century. Because the fluid flow in straight pipes and channels resembles in many ways the motion of air near the surface of the earth a large part of the present wind profile theory for the surface boundary layer was developed from these investigations.

Reynolds (1883) conducted a series of experiments on flow in long straight pipes. By introducing colouring matter into water flowing through glass pipes he demonstrated the existence of two widely different modes of motion. The motion was laminar in character at low velocities and turbulent (or eddying) at high velocities. The change from laminar to turbulent motion took place suddenly at a critical velocity which was directly proportional to the kinematic viscosity (ν) of the fluid and inversely proportional to the diameter (d) of the pipe. Reynolds concluded that the transition from laminar to turbulent flow depends only on the value of a dimensionless ratio given by:

$$Re = \frac{Ud}{\nu} \tag{2.1}$$

Here U is the average velocity and $\nu = \mu/\rho$, where μ is the dynamic viscosity and ρ is the density. This ratio became known as the Reynolds number and is a fundamental quantity in fluid mechanics.

The Reynolds number has been the subject of many subsequent investigations both practical and theoretical. It was found that in a viscous flow the viscous forces can be represented as $\mu U/L^2$ and the inertia forces can be characterized as $\rho U^2/L$, where L is any characteristic length. The ratio of the inertia to the viscous forces is identified as the Reynolds number, giving the relative importance of the two forces.

2.2 The Richardson Number

While the Reynolds number was particularly useful in defining the transition from laminar to turbulent flow in pipes, where a density gradient is absent, it proved to be less useful in the atmosphere where a density gradient is generally present. It was found that the existence of the buoyancy forces near the earth's surface has a major effect on the atmospheric turbulence. According to Businger (1955), the important distinction in the surface boundary layer is not between the laminar and the turbulent flow but rather between turbulence caused by mechanical friction and turbulence caused by convection. For example, in the neutral atmosphere turbulence near the earth's surface is entirely due to friction. As soon as a heat flux exists turbulence will either increase or decrease through the liberation or absorption

of convective energy.

Richardson (1920) postulated that the ratio of the rate at which buoyancy forces extract energy from turbulence to the rate at which it is supplied by the shear stress determines whether turbulence will increase or decrease. This ratio became known as the flux Richardson number (R_f) and is a fundamental quantity in micrometeorology.

Lumley and Panofsky (1964) write the flux Richardson number as:

$$R_f = \frac{g}{\overline{\theta}} \frac{\overline{w\theta}}{\overline{uw}\ (\partial U/\partial z)} \tag{2.2}$$

where the quantities $\overline{w\theta}$ and $\overline{uw}$ can be defined in terms of the exchange coefficients for heat (K_H) and for momentum (K_M) as:

$$K_H = \frac{-\overline{w\theta}}{\partial\theta/\partial z} \quad \text{and} \quad K_M = \frac{-\overline{uw}}{\partial U/\partial z} \tag{2.3}$$

where $\overline{\theta}$ is the mean potential temperature. In theory, if R_f is equal to unity then the buoyancy forces remove energy as fast as it is introduced by the shear, and the turbulence cannot maintain itself. However, this does not mean that $R_f = 1$ gives the criterion for the onset or disappearance of turbulence.

By substituting Eqs. (2.3) into (2.2) R_f may be written as:

$$R_f = \frac{K_H}{K_M} Ri \tag{2.4}$$

where Ri is known as the gradient Richardson number and it is defined as:

$$Ri = \frac{g}{\theta} \frac{\partial \theta / \partial z}{(\partial U / \partial z)^2} \tag{2.5}$$

Since the ratio K_H/K_M is not readily obtained from experimental data the gradient Richardson number (Ri), which can be identified as the ratio of buoyancy to inertia forces, is generally used in practice. It was shown by Batchelor (1953) that Ri is a relevant parameter for the specification of turbulent flow near the earth's surface under non-adiabatic conditions. The chief advantage of Ri is that it can be estimated from measurements of winds and temperatures at two levels. Observations have shown that Ri can be used as an index of stability but it has a disadvantage in that it varies with height and like R_f it fails to predict the onset or disappearance of turbulence. Nevertheless, Ri has many practical applications and will be discussed further in connection with the log-linear law.

Stability analyses indicated that there exists a critical Richardson number at which turbulence is damped out. However, the

correct value of this critical number is uncertain. Townsend (1958) and Kao (1959) found that the theoretical upper and lower limits for the critical flux Richardson number are 0.5 and -0.5 respectively. Laboratory measurements showed that in a stably stratified viscous boundary layer turbulence would not grow when $Ri > 0.0417$ (Schlichting, 1968). Furthermore, measurements in the atmosphere indicated that turbulence is not found when $Ri > 0.2$ (Lumley and Panofsky, 1964). Attempts were made by Rossby and Montgomery (1935), Lettau (1949), Businger (1955) and others to replace Ri by a parameter which would give a more complete description of turbulence. However, thus far, no entirely satisfactory parameter has been found.

CHAPTER III

WIND PROFILE MODELS

3.1 The Wind Profile Observations and the Power Law

At the beginning of the twentieth century extensive experimental results for flow in smooth and rough pipes were obtained by Burgers, Zijnen, Hansen, and Nikuradse and summarized by Schlichting (1968). Based on these results it was found empirically that the velocity distribution in fully developed turbulent flow can be represented by a power law in the form:

$$U \propto z^p \tag{3.1}$$

where z is the distance from the boundary and p is an empirically determined exponent. Von Karman and Prandtl (Brunt 1934, p. 236; Binder 1955, p. 122) found that $p = 1/7$ for $Re < 5 \times 10^4$ in smooth pipes and decreases to $p = 1/10$ for $Re \simeq 10^6$ in rough pipes.

Experiments to determine the velocity of the air close to a solid boundary began with Stanton (1911). Using a pitot tube he measured the air velocities to within 0.001 inches from the solid boundary and obtained a number of velocity profiles. In 1921 von Karman analysed Stanton's observations and found that they fitted the one-seventh power law $(p = 1/7)$ with great accuracy.

The discovery that the velocity of the fluid flow in pipes can be represented by a power law prompted many meteorologists to

apply the power law to the measurements of the air flow near the earth's surface. As early as 1908 Akerblom studied the variation of wind speed with height using observations taken on the Eiffel Tower. Furthermore, he used the change in the wind direction between the top and the bottom of the Eiffel Tower to find the coefficient of viscosity of the atmosphere due to turbulence.

The variation of wind speed with height became of increasing interest after the development of the theory of atmospheric turbulence by Taylor (1915a). Taylor (1915b) used pilot balloon observations, obtained over Salisbury Plain, England, in 1914, to calculate the skin friction of the wind blowing over the earth's surface. He expressed the skin friction (F) on unit area of the earth's surface as:

$$F = k_s \rho Q_s^2 \tag{3.2}$$

where Q_s is the wind velocity near the surface, ρ is the density of the air and k_s is a constant which depends on the surface roughness and is determined from the wind profiles. For the ground at Salisbury Plain the value of k_s was found to be between 0.002 and 0.003.

The wind profile data were frequently required in the military operations, particularly in the chemical and biological warfare and in the problem of screening targets by smoke. Taylor (1915c), with the help of Cave, published numerous tables showing the wind velocities

at 1.25, 4 and 6 ft above ground at various stations. They wrote:

> "The observations described in the following pages were undertaken at the request of the Director of the Meteorological Office with a view to providing information for the use of the officers responsible for discharging of gas during a gas attack".

According to Frost (1948), Hellmann in 1919 obtained wind profile data from recording anemometers set up at 5, 25, 50, 100 and 200 cm above ground and fitted the observations to the power law in the form:

$$U = U_1 \left(\frac{z}{z_1}\right)^p \tag{3.3}$$

where U is the mean wind velocity at height z and U_1 is the mean wind velocity at a constant reference height z_1. Hellmann found that $p = 0.27$ when all of the observations were used and $p = 0.33$ when only the night observations were used.

Geiger (1927) suggested that the value of the exponent p is not constant but depends principally on height. He reasoned that because with increasing height the effect of ground friction diminishes the value of p becomes smaller.

Heywood (1931) obtained wind velocities from two anemometers at heights of 12.7 and 94.5 m, above ground, over a period of two years. He found that the wind gradient between two heights close to the ground depends on the wind velocity of the upper of the two layers, the vertical temperature gradient, and the nature of the ground.

Fitting the observations to the power law of the form used by Hellmann, Heywood found that $p = 0.26$, which was in good agreement with Hellmann's value.

Sutton (1932) emphasized that the value of p depends to a large extent on the vertical temperature gradient, a point which had previously been neglected. He used Heywood's observations and found that during the summer the change in p from midday to midnight was from 0.07 to 0.17, while during the winter the corresponding change in p was from 0.08 to 0.13. Although, the change in p from summer to winter was slight it did show that p is least during the period of the strongest lapse and greatest during the period of the largest inversion.

More conclusive results concerning the variation of p with stability were obtained by Barkat Ali (1932). He fitted the power law to wind measurements taken at Agra, India at heights of 1.7 and 21.7 m above ground. The values of p were obtained separately for winter (Dec. - Jan.), hot season (Apr. - May) and monsoon season (Jul. - Aug.). It was found that during each season the value of p was largely determined by the existing lapse rate of temperature. In hot and monsoon seasons the value of p ranged from 0.0 with large positive lapse rates in the middle of the hot afternoon to 0.46 with extreme inversions at night. In the mid-winter season values of p as high as 0.87 occurred towards midnight.

Further studies to determine the effect of lapse-rate of temperature on the value of p were carried out by Giblett (1935).

He grouped 50 ft and 150 ft wind speeds according to the temperature difference (ΔT) between 143 ft and 4 ft. It was found that the ratio U_{150}/U_{50} increased with increasing stability from a value of 1.0 when $\Delta T = -5$ F to a value of 1.17 when $\Delta T = 0$ F and to a value of 1.98 when $\Delta T = 7$ F. The power law was fitted to the U_{150}/U_{50} ratios for each ΔT and the corresponding values of p were found to be 0.001, 0.143 and 0.630 respectively.

Best (1935) studied the increase in wind velocity in relation to simultaneously occurring temperature gradients and found that the temperature gradient and the wind gradient are interrelated.

In addition to dependence of p on temperature gradient, Paeschke (1937) found that p increases with increasing roughness of the surface. He carried out an experimental study and found that p varied from 0.20 over smooth snow surface with roughness length $z_o = 3$ cm to 0.33 over a turnip field with $z_o = 45$ cm. It was also found that p depends not only on the height, but also on the kind of plant cover. For example, over a wheatfield with $z_o = 130$ cm, it was found that $p = 0.29$, while over a turnip field with $z_o = 45$ cm, $p = 0.33$.

Carruthers (1943) summarized the relevant literature concerning the work that was done on variation of wind with height until that time. The summary of the result is shown in Table 3.1.

Table 3.1

Values of p found by different investigators

Observation heights (m)	Value of p	Source
0 to 123	0.17 to 0.33	G. Hellmann 1917, 1919
0 to 2	0.10 to 0.44	A.C. Best 1935
Various heights	0.14 to 0.25	R. Geiger 1927
1.7 to 21.7	0.0 to 0.46 to 0.87 for large inversions	Barkat Ali 1932
Height not stated	0.13	F.J. Scarse
11 to 30	0.17	J.S. Dines
13 to 95	0.08 to 0.17	O.G. Sutton (Heywood's data) 1932
16 to 46	0.009 to 0.62	M.A. Giblett 1932
Height not stated	0.009 to 0.62	O.G. Sutton 1934

During the 1943-1945 period wind profile observations, up to a height of 400 ft, were taken at Cardington, England. The measurements were made at 5, 25, 50, 100, 200 and 400 ft above ground with instruments suspended from a balloon. Frost (1947) analysed these observations and showed that if the wind observations were grouped with respect to temperature difference between 400 ft and 5 ft then the mean winds fitted the power law and p was approximately a linear function of the temperature gradient. Further measurements of wind

speeds up to a height of 1000 ft were made at Cardington during 1946. Analysing these data Frost (1948) considered only wind speed profiles for which the temperature difference between 4 ft and 1000 ft was between 5 F and 6 F and when the wind speed at any height between 4 ft and 1000 ft was greater than 20 mph. He plotted logarithms of mean velocities of 64 observations at 8 heights against logarithms of heights and found that the points lie on a straight line having the slope equal to 0.149. Frost concluded that a value of $p = 1/7$ is applicable to an atmosphere with a dry adiabatic lapse rate over the first 1000 ft, while $p = 1/2$ may be taken as appropriate for inversions.

Johnson (1959) analysed wind profile observations obtained at Suffield Experimental Station, Alberta, and at O'Neill, Nebraska. The Suffield wind profiles, extending from 0.5 - 16 m, were measured over prairie grass and over snow surface while the O'Neill wind profiles were taken over short grass and extended from 2 to 16 m. He found that over the snow surface under near-adiabatic conditions $p = 0.165$. Furthermore, it was found that the value of p increased with increased surface roughness and with increased stability. The maximum values of p for Suffield and for O'Neill were found to be 0.75 and 0.4 respectively.

A comprehensive study of wind speed profiles was completed by DeMarrais (1959). The data for this study were from the 125-m meteorological tower at Brookhaven National Laboratory. The study showed that the value of p decreased as the lapse rate changed

from marked stability to slight instability. Thereafter, the value of p leveled off under moderate superadiabatic conditions and then rose again in strongly superadiabatic conditions. It was also shown that the value of p was dependent on the wind speed and direction. The parameter p decreased with increased wind speed and it appeared that p was relatively smallest in the quadrant in which the wind had the longest unhindered fetch over the smoothest terrain. There was also an indication that the value of p decreased with height. The uppermost layers had the smallest p-values during unstable conditions while the value of p was highest when the lapse rate was less than adiabatic.

Panofsky, Blackadar and McVehil (1960) considered wind profiles in the height range of 11 - 46 m and produced a nomogram relating the power p to the roughness length z_o and to the inverse of the Monin-Obukhov length L. The nomogram, which was intended primarily for engineering applications, shows that the value of p increases with increasing values of z_o and decreases with increasing values of 1/L.

Munn and Richards (1963) obtained the median values of p for each hour of the day from the 20-ft and 80-ft winds at Douglas Point, Canada. These p-values were plotted against time and the graph shows that the value of p was between 0.2 and 0.3 during the day and between 0.5 and 0.7 at night.

Pettitt and Root (1965) examined wind measurements obtained from the 200-ft micrometeorological towers located at Montreal, Quebec,

and at Whiteshell, Manitoba. At Montreal, 35-ft and 200-ft wind speeds were considered in the analysis. The ranges of daytime p-values for winter, spring, and fall seasons were found to be 0.13 - 0.30, 0.14 - 0.34 and 0.15 - 0.32 respectively, while the corresponding ranges for night-time p-values were 0.14 - 0.36, 0.21 - 0.35 and 0.17 - 0.37. At Whiteshell 20-ft and 200-ft wind speeds were considered and the values of p ranged from 0.18 - 0.20 in summer and from 0.12 - 0.16 in winter.

Panofsky and Prasad (1965) showed that the power law exponent p given by:

$$p = \frac{\partial(\ln U)}{\partial(\ln z)} \tag{3.3a}$$

is a universal function of z/z_o and z/L' where L' is the gradient length and will be discussed in some detail in section 3.4. They found that the exponent p at a fixed height increases with roughness and with stability. Other studies were done to investigate the variations in p and will be discussed further in Chapter IV.

3.2 The Logarithmic Law

While the power law representation of the variation of wind with height gave reasonably good results it had no theoretical foundation. The theoretical analyses of the phenomenon based on the equations of motion of fluids started with Prandtl in 1904. In his now classical paper presented to the Third International Congress of Mathematicians held in 1904 in Heidelberg Prandtl showed that for

a fluid of small viscosity, such as air or water, the viscosity will substantially affect the flow only in a thin layer adjacent to the surface. Prandtl called this layer the "Grenzschicht" (boundary layer). The boundary layer theory was extended after 1904 by Prandtl himself, von Karman, and others. By introducing a mixing length, which was assumed to be proportional to the height above ground, and using dimensional analysis, Prandtl was able to show that the velocity distribution in the turbulent boundary layer can be expressed by a logarithmic law. Von Karman (1954) wrote: "The formulation of the logarithmic law was the end result of a long struggle to obtain correlation between theoretical ideas and experimental evidence".

Although, the logarithmic law was primarily intended for studies of flow in pipes it was soon applied to the wind profile observations close to the ground. According to Frost (1948), Chapman in 1919 found that the mean wind speed observations in the layer from 10 to 500 m were represented well by a logarithmic law in the form:

$$U = A + B \log z \tag{3.4}$$

where A and B are constants. In the same year Hellmann proposed another logarithmic law in the form:

$$U = A + B \log (z + C) \tag{3.5}$$

where A, B and C are constants which vary with location and with time.

Rossby and Montgomery (1935) found that in an adiabatic atmosphere the wind velocity could be represented by the logarithmic law in the form:

$$U = A\left[\log(z + z_o) - \log z_o\right] \qquad (3.6)$$

where A is a constant and z_o is a length which depends on the surface roughness. Over open grassland it was found that z_o = 3.2 cm.

Sutton (1953) gives the following forms of the logarithmic law based on Prandtl's theoretical work:

For smooth flow: $$U = \frac{u_*}{k} \ln \left(\frac{u_* z}{\nu}\right) + \text{constant} \qquad (3.7)$$

For fully rough flow: $$U = \frac{u_*}{k} \ln \frac{z}{z_o} \qquad (3.8)$$

For very rough surfaces: $$U = \frac{u_*}{k} \ln \left(\frac{z-d}{z_o}\right), \quad z \geq d + z_o \qquad (3.9)$$

where $u_* = (\tau_o/\rho)^{\frac{1}{2}}$ is the friction velocity, τ_o is the shearing stress at the surface, z_o is the roughness length, ν is the kinematic viscosity, k is von Karman's constant and d is the zero-plane displacement which is regarded as a datum level above which the normal turbulent exchange takes place.

Thornthwaite and Halstead (1942) plotted the first and the second power of the wind velocity against the logarithm of height

and found that for night-time observations the first power of velocity was satisfactory to obtain a linear variation while for day-time observations the square of the velocity gave a better fit. Consequently, they formulated a wind law which was a combination of the logarithmic and the power law. This new law assumed the form:

$$U = \left[(\log z - \log z_o) \;/\; \log a\right]^{1/P} \tag{3.10}$$

where $\log z_o$ is the y-intercept and $\log a$ is the slope of a straight line obtained by plotting $\log z : U$. The value of P was believed to vary between 2.0 with fully developed turbulence and some value less than 1.0 when turbulence reached its smallest actual value.

Thornthwaite and Kaser (1943), using newly designed instruments, made precise wind profile measurements in the 0.5 - 28 ft layer. The data showed that the graph of $\ln z : U$ was a straight line during the morning and evening hours when the lapse rate was adiabatic. On the other hand, when the thermal structure was superadiabatic the graph was convex to the U-axis, and for inversions the curvature of the graph became concave.

Halstead (1943) studied the $\ln z : U$ graphs and, by introducing a stability term, extended the logarithmic law to fit the wind profile observations made under all stability conditions. The modified logarithmic law, proposed by Halstead, has the form:

$$U_2 - U_1 = \frac{u_*}{k} \ln \frac{z_2}{z_1} + \frac{c\alpha p}{T_o} (z_2 - z_1) \tag{3.11}$$

where α_p is the lapse rate of potential temperature, c is a constant and T_o is the surface temperature. The stability term $(c\alpha_p/T_o)(z_2 - z_1)$ is positive for stable air, negative for unstable air, and zero for neutral equilibrium. To test the validity of the modified logarithmic law Halstead used part of the data obtained by Thornthwaite and Kaser and showed that the graph of $\ln(z_2/z_1)$: $[(U_2 - U_1) - c\alpha_p/T_o)(z_2 - z_1)]$ was, very nearly, a straight line for all stability conditions.

3.3 The Deacon's Generalized Wind Profile Law

A significant contribution to the formulation of the wind profile laws was made by Deacon (1949). Following an extensive investigation of the wind profiles in the lowest eight meters of the atmosphere Deacon proposed the generalized wind profile law in the form:

$$\frac{dU}{dz} = az^{-\beta} \qquad (3.12)$$

where U is the horizontal wind speed at height z, and a and β are constants in any given situation. The parameter β, which reflects the degree of curvature of the wind speed profile is a function of stability and assumes values: $\beta > 1$ for unstable conditions, $\beta = 1$ at neutral stability and $\beta < 1$ for stable conditions. In the integrated form the Deacon's generalized wind profile law becomes:

$$U = \frac{u_*}{k(1-\beta)}\left[\left(\frac{z}{z_o}\right)^{1-\beta} - 1\right], \quad \beta \neq 1 \qquad (3.13)$$

For neutral conditions $\beta = 1$; and the equation (3.12) transforms into the logarithmic law. The parameter β, which is regarded as a measure of stability was evaluated by a number of investigators and several methods for its evaluation were developed. Deacon found that for velocity profiles over long grass the value of β varied from 1.2 under superadiabatic conditions to 0.75 under marked inversions. The values of β were obtained from the equations:

$$\frac{U_2}{U_1} = \frac{\left(\frac{Z_2}{z_o}\right)^{1-\beta} - 1}{\left(\frac{Z_1}{z_o}\right)^{1-\beta} - 1} \tag{3.14}$$

and

$$\frac{U_3 - U_2}{U_2 - U_1} = \frac{Z_3^{1-\beta} - Z_2^{1-\beta}}{Z_2^{1-\beta} - Z_1^{1-\beta}} \tag{3.15}$$

where U_1, U_2, U_3 are wind speeds at heights Z_1, Z_2, Z_3 respectively and z_o is the roughness length.

Rider (1954) used wind speeds at 37.5 cm and 150 cm, and a surface roughness of 0.32 cm, and found that the value of β ranged from 1.13 to 0.78. Furthermore, Rider discovered that the Deacon's generalized wind profile law was satisfactory under unstable conditions only and in order that the law would fit the observations under stable conditions, z_o would have to increase in magnitude with increasing stability.

Longley (1956) evaluated the parameter β using the relationship:

$$\beta = 1 - \frac{\log \dfrac{U_3 - U_2}{U_2 - U_1}}{\log r} , \qquad r = \frac{Z_3}{Z_2} = \frac{Z_2}{Z_1} \tag{3.16}$$

where U_1, U_2, U_3 are wind speeds at heights Z_1, Z_2, Z_3 respectively. The values of β were computed from several samples of data including data obtained at Suffield Experimental Station. A comparison of the results showed that it was impossible to obtain β with sufficient accuracy to permit clear distinction between different types of stability and this led to the conclusion that its use in any practical problem was not warranted.

Davidson and Barad (1956, 1957) analysed the data obtained at O'Neill, Nebraska and found that the parameter β varied with height. The variation was such that β decreased with height under stable conditions and increased with height under unstable conditions. They concluded that, " ... the Deacon formula, although adequate for gross wind interpolation purposes, does not accurately describe the effect of stability variation on the wind profile in the first six meters above the ground".

Palmer (1956) calculated the values of β by using Barad, Craw, and Longley methods. The comparison of results showed that the values of β, obtained by the Craw and Longley methods, were almost identical for the same heights. No direct comparison was

possible with Barad's method since it involved the use of wind data from different heights. It was concluded that the Longley method was superior to the Craw method since the former involved less computation and had less chance of an arithmetic error.

Deacon (1956) suggested a reason for the failure of his generalized wind profile law to predict correctly the shearing stress under certain conditions. He found evidence that there was a marked rise in the drag coefficient as the wind speed decreased to a low value and this applied both to stable and unstable conditions. Consequently, there was an increase in the drag coefficient with decreasing Reynolds number. Deacon concluded that the source of the reported failures of his generalized wind profile law was due to the absence of fully rough flow.

In addition to evaluations of the parameter β several investigations were made to examine the parameters u_*, z_o and d. Different methods for evaluating these parameters were given by Deacon (1949), Panofsky (1952, 1963), Kao (1959), Robinson (1962) and others.

Deacon (1949) postulated that at sufficiently high fluid velocities the roughness parameter z_o is dependent only on the form and distribution of the roughness elements while at much lower fluid velocities the roughness elements are submerged in a shallow viscous sub-layer adjacent to the surface and $z_o = \nu/ (9.05\ u_*)$. Deacon's observations over long grass showed a marked decrease of z_o with increasing wind velocity and he concluded that the reason for this

decrease in z_o was due to the long grass blades bending to the wind to produce a smooth surface. Based on Deacon's work Sutton (1953) prepared a table of z_o and u_* for natural surfaces. Sutton's table shows that z_o varies from 0.001 cm over mud flats to 9 cm over thick grass up to 50 cm high, while u_* varies from 16 cm sec^{-1} to 63 cm sec^{-1} for the same two surfaces respectively.

Hage (1961) evaluated z_o from wind profile measurements obtained at Suffield Experimental Station and found that over sparse prairie grass z_o increased with decreasing wind speed. The range of z_o was from $z_o = 0.7$ cm at $U_{0.5} = 6.45$ m sec^{-1} to $z_o = 7.4$ cm at $U_{0.5} = 1.00$ m sec^{-1}.

Dickson and Allbee (1967) evaluated the friction velocity u_* for 1370 wind profiles using procedure outlined by Kao (1959). They found that the friction velocity is primarily affected by the velocity shear but not affected by thermal stratification. Thus they concluded that u_* can be used in any atmospheric stability condition.

3.4 The Log-Linear Law

Most of the recent progress in the understanding of the variation of wind and temperature with height was brought about by Monin and Obukhov (1954). They introduced the, now well-known, similarity theory according to which there exist near the ground a velocity u_*, a length L and a temperature T* that are essentially invariant with height. Under the conditions of similarity the wind profile in the surface boundary layer can be expressed as a function

of the non-dimensional height z/L in the form:

$$S = \phi \tag{3.17}$$

where S is a non-dimensional wind shear defined by:

$$S = \frac{kz}{U_*} \frac{\partial U}{\partial z} \tag{3.18}$$

and $\phi = \phi(z/L)$ is a universal function. The Monin-Obukhov length L may be defined as:

$$L = - \frac{\rho U_*^3 C_p T}{k g H} \tag{3.19}$$

where T is air temperature, C_p is specific heat a constant pressure, H is vertical heat flux and the other symbols have their usual meaning. Once the function ϕ is specified, Eq. (3.18) can be integrated.

Considerable effort has been made over the last few years to find the universal function ϕ and several functional relationships were proposed.

Monin and Obukhov (1954) suggested that for near-adiabatic conditions the function ϕ can be expanded in power series as:

$$\phi = [1 + \alpha_1 (z/L) + \alpha_2 (z/L)^2 + \cdots] \tag{3.20}$$

then for small values of z/L the function can be approximated by:

$$\phi = 1 + \alpha(z/L) \tag{3.21}$$

where α is a constant to be determined from observations under near-neutral conditions. Substituting the function given by Eq. (3.21) into Eq. (3.17) and integrating yields the familiar Monin-Obukhov log-linear law:

$$U = \frac{u_*}{k}\left[\ln\frac{z}{z_o} + \alpha\frac{z}{L}\right], \quad z \gg z_o \tag{3.22}$$

Since the Monin-Obukhov length L can be measured only when the vertical heat flux is known, which is rarely the case, Panofsky, Blackadar and McVehil (1960) introduced a gradient length L' defined as:

$$L' = \frac{u_* \theta\ (\partial U/\partial z)}{kg\ (\partial\theta/\partial z)} \tag{3.23}$$

where θ is the potential temperature and the other symbols have their usual meaning. The gradient length L' is related to L by:

$$L' = \frac{K_H}{K_M} L \tag{3.24}$$

where K_H and K_M are exchange coefficients for heat and momentum respectively. The question of the relative sizes of K_H and K_M has still not been answered satisfactorily. However, investigations

suggest that the ratio K_H/K_M is not too far from unity (Lumley and Panofsky 1964).

Assuming that $K_H/K_M \simeq 1$ the modified Monin-Obukhov function can be written as:

$$\phi = 1 + \alpha(z/L') \tag{3.25}$$

and

$$U = \frac{u_*}{k} \left[\ln \frac{z}{z_o} + \frac{\alpha z}{L'} \right], \quad z >> z_o \tag{3.26}$$

Another functional relationship which was found particularly useful for specifying the function ϕ was derived in different ways by Kazansky and Monin (1956), Ellison (1957), Yamamoto (1959), Panofsky (1961), and Sellers (1962), and has the form:

$$\phi^4 - 18 \left(\frac{z}{L'}\right) \phi^3 = 1 \tag{3.27}$$

This relationship became known as the KEYPS function after the initials of its inventors.

Priestley (1960) advanced the view that the values of the Monin-Obukhov coefficients (α's) are products of smoothing rather than true physical constants of a steady regime intermediate between free and forced convection. He defined a junction height between free and forced convection as:

$$\frac{z}{L} = - k^4 h^{-2} (K_H/K_M)^3 \tag{3.28}$$

where for $k = 0.4$, $h = 0.9$ and $K_H/K_M = 1$, the junction height $z/L = -0.0316$ at the unsmoothed junction. Furthermore, Priestley specified the function ϕ for free convection as:

$$\phi = \frac{K_H}{K_M} h^{-2/3} k^{4/3} \left| \frac{z}{L} \right|^{-1/3} \tag{3.29}$$

where h is the Priestley constant.

Webb (1960) suggested a two-part function ϕ with continuous first, second, and third derivatives at the junction. This function is of the form:

$$\phi = 1 + 4.5(z/L') \qquad \text{for } -z/L' < 0.0317 \tag{3.30}$$

and

$$\phi = 0.316(-z/L')^{-1/3} - 0.00143(-z/L')^{-4/3} \quad \text{for } -z/L' \geq 0.0317 \tag{3.31}$$

Businger (1961) studied the spectrum of atmospheric turbulence and proposed a universal function in the form:

$$\phi^{8/3}\{ -R_f^{\,2/3} \left[\left(\frac{K_m}{K_c}\right)^{8/3} - 1 \right] + (-R_f + 1)^{2/3} \} = 1 \tag{3.32}$$

where $K_m/K_c = 1.7$ and R_f is the flux Richardson number.

Panofsky (1963) showed that, in general, the log-linear law can be written in the form:

$$U = \frac{u_*}{k} \left[\ln \frac{z}{z_o} - \psi(z/L') \right] \tag{3.33}$$

where $\psi(z/L')$ is another universal function which is related to $\phi(z/L')$ by:

$$\psi(z/L') = \int_{o}^{-z/L'} \frac{1 - \phi(\xi)}{\xi} \, d\xi \tag{3.34}$$

Brooks et al (1963), following Sheppard's (1958) arguments, found that a better fit to observed wind profiles was possible by applying a square root modification to the Monin-Obukhov universal function and defined a new universal function as:

$$\phi = \left[1 + \gamma_1 \left(\frac{z'}{L'}\right)^{\frac{1}{2}} \right] \tag{3.35}$$

where $z' = z - d$ and $z' \geqslant z_o$. In the integrated form the Brooks log-power law becomes:

$$U(z') = \frac{u_*}{k} \left[\ln \frac{z'}{z_o} + 2\,\gamma_1 \left(\frac{z'}{L}\right)^{\frac{1}{2}} \right] \tag{3.36}$$

where γ_1 was found to be 1.24.

Swinbank (1964) introduced a new non-linear height variable X which had the property that the graph of U: log X was a straight line no matter what the stability. He then derived a universal function in the form:

$$\phi = \frac{Z}{L} \left[1 - \exp(-Z/L) \right]^{-1} \tag{3.37}$$

which upon integration yields the Swinbank exponential wind profile:

$$U_2 - U_1 = \frac{u_*}{k} \ln \left[\frac{\exp(Z_2/L) - 1}{\exp(Z_1/L) - 1}\right] \tag{3.38}$$

where the symbols have their usual meaning.

McVehil (Lumley and Panofsky 1964) found that in stable air for $Z/L' \leqslant 0.3$, $(Ri \leqslant 0.1)$, the log-linear law assumes the form:

$$U = \frac{u_*}{k} \left[\ln\frac{z}{z_o} + \beta z \right] \tag{3.39}$$

where $\beta = 1/Ri - L'/Z$. However, when $Z/L' > 0.3$ no simple relationship exists for the universal function ϕ. Furthermore, it was suggested that at large values of Ri turbulence changes character and is gradually replaced by gravity waves.

Brooks, Mulholland and Smith (1965) experimented with a hyperbolic and an inverted exponential form of the universal function. This resulted in a log-law plus power law binomial given by:

$$U(Z') = \frac{u_*}{k} \ln\frac{Z'}{z_o} + c\left(\frac{Z'}{Z_h}\right)^b, \quad z_o \leqslant Z' \leqslant Z'_h \tag{3.40}$$

where Z_h is a characteristic height, b is the exponent and c is a constant.

Pandolfo (1966) established the form of the universal function in the constant flux boundary layer in lapse stratification. He found

that Monin-Obukhov function applies in lapse forced convection conditions and Priestley's function applies in the free convection conditions. The integrated wind profile laws for forced and free convection then become:

$$U(Z_2) - U(Z_1) = \frac{u_*}{k} \left[\ln \frac{Z_2}{Z_1} + \frac{\alpha}{L} (Z_2 - Z_1) \right] \quad \text{for } 0 \geq \frac{Z}{L} \geq \left(\frac{Z}{L}\right)_\tau \tag{3.41}$$

$$U(Z_2) - U(Z_1) = \frac{u_*}{k} \, 6 \left(\frac{c}{3}\right)^{\frac{1}{2}} \left[\left|\frac{Z_1}{L}\right|^{-1/6} - \left|\frac{Z_2}{L}\right|^{-1/6} \right] \text{for } (Z/L)_\tau \geq Z/L \tag{3.42}$$

where $c = 3k^{4/3} h^{-2/3}$ and h is the Priestley constant.

With the introduction of the log-linear law it became necessary to determine the constants which enter into the equation. In addition to friction velocity u_* and roughness length z_o, which entered into the logarithmic law, the log-linear law requires the Monin-Obukhov constant α and the length L. In the Monin-Obukhov original analysis it was found that $\alpha = 0.6$. However, it was shown by a later study (Taylor 1960) that the value of α is about six. The reason for this discrepancy was due to the fact that Monin and Obukhov applied the log-linear law to a larger range of stability than was justified.

Taylor (1960) calculated the values of α from the data given by Monin and Obukhov, Swinbank, and Rider and obtained different values of α for forced convection, free convection and inversion conditions. The numerical results obtained by Taylor are given in Table 3.2. He concluded that the value of α is about six for

stability corresponding, approximately,, with forced convection conditions.

Table 3.2

Values of α

	Rider	Swinbank	Monin-Obukhov
Forced convection	11.7 ± 1.5	6.1 ± 1.5	6.0 ± 0.9
Free convection	0.85± 0.03	1.43± 0.14	1.31± 0.03
Inversion	2.4 ± 0.3	5.8 ± 1.3	--

Priestley (1960) found that the values of the Monin-Obukhov constants in superadiabatic conditions can be predicted by the equation:

$$\alpha_i = (\gamma^3 k^4 h^{-2})^{-i} \frac{(0.7121)^i}{(3i+1)i!} \tag{3.43}$$

where $\gamma = K_H/K_M$, k = von Karman's constant, h = Priestley's constant and $i = 1, 2, 3, \ldots$. The numerical values of α_i then become $\alpha_1 = 5.63$, $\alpha_2 = 36.3$, $\alpha_3 = 191$, and so on. When the coefficients given by Eq. (3.43) are substituted into Eq. (3.20) the function ϕ consists of terms of alternating sign since L is negative for superadiabatic conditions. Consequently, the function ϕ converges for all finite z.

McVehil (1964) analysed wind and temperature profiles from O'Neill, Nebraska, and Antarctica and found that in stable air the value of α is approximately seven.

CHAPTER IV

ANALYSES AND RESULTS

4.1 The Power Law

The current analyses began with the determination of the power law exponent p. By differentiating the power law given by Eq. (3.3) logarithmically it can be shown that, in the finite difference form, the power law exponent is given by:

$$p = \frac{\Delta \ln U}{\Delta \ln z} \tag{4.1}$$

By plotting $\ln z : \ln U$ and using the least squares method the value of p was calculated for each profile. In addition, to determine the accuracy with which the observed data are represented by the power law the correlation coefficient between $\ln z$ and $\ln U$ was determined for each profile. The logarithm of height was considered as the independent variable. The application of the correlation coefficient will be discussed further in the next chapter.

In order to compare the values of p obtained in this study with the values obtained by different investigators the method of comparison used by DeMarrais (1959) was adopted. The wind profiles were grouped into the following categories: superadiabatic, adiabatic, stable and inversion. The average value of p was then obtained for each category. The values of p obtained in this investigation are compared with those obtained by other investigators in Table 4.1.

Table 4.1

A comparison of the values of p obtained in this study with those obtained at other locations

Site	Height Range (m)	Terrain	Super-adiab.	Neutral	Stable	Inversion
* Suffield (41 profiles)	0.5-16	sparse prairie grass	0.22	0.23	--	0.31
* Suffield (129 profiles)	2 - 92	sparse prairie grass	0.18	0.15	0.17	0.32
* Suffield (129 profiles)	16 - 92	sparse prairie grass	0.09	0.14	0.19	0.36
* Kerang, Australia	0.5-16	grazing land	0.11	--	--	--
Suffield(1959)	0.5-16	prairie grass	--	--	0.22	0.49
Suffield(1959)	0.5-16	snow	0.16	0.17	--	0.28
O'Neill, Neb.	2 - 16	short grass	0.11	--	0.17	0.38
Quickborn, Germany	10 - 70	meadows	0.25	0.27	--	0.61
Tallmadege, Ohio	11 - 49	flat field	0.16	0.20	0.25	0.36
Hanford, Washing.	15 -122	mountainous	0.09	0.12	0.14	0.25
Cardington, England	8 -120	grass field	0.15	0.17	0.27	0.32 to 0.77
Harwell, England	9 - 27	airfield	0.09	0.08	0.18	--
Idaho Falls, Idaho	6 - 61	desert	0.15	0.18	0.22	--
Brookhaven	11 -124	nearby wooded area	0.19	0.29	0.35	0.46 to 0.59
Brookhaven	11 -125	nearby wooded area	0.19	0.28	0.38	0.51
Brookhaven	46 -125	nearby wooded area	0.18	0.31	0.45	0.59
Brookhaven	11 - 46	nearby wooded area	0.23	0.34	0.34	0.37

* Data obtained in the present study

A study of the results in Table 4.1 reveals that the value of p depends on the location, stability, height range, and surface roughness. It is observed that the values of p obtained for Suffield 0.5-16 m layer by Johnson (1959) are in fairly good agreement with those obtained in the present study. The small differences which do exist may arise from the fact that Johnson calculated the values of p for a specific temperature gradient while in the present study average values of p are given. In the Suffield 2-92 m layer the values of p appeared to increase in extreme superadiabatic conditions. Consequently, a relatively high average value of p was obtained for lapse conditions. Such rise in the values of p under extreme superadiabatic conditions was also reported by DeMarrais (1959).

Investigations of wind profiles carried out at the Brookhaven National Laboratory indicated that, in addition to the temperature gradient, height range and surface roughness, the value of p varies with the vertical wind shear. In order to account for the variation of wind with height and to obtain a measure of convective stability the gradient Richardson number was calculated for each profile. The calculations were performed using the gradient Richardson number in the finite difference form; defined as:

$$Ri = \frac{g(\Delta\theta/\Delta z)}{\theta(\Delta U/\Delta z)^2} \tag{4.2}$$

The potential temperature θ was calculated from Poisson's equation in the form:

$$\theta = T \left(\frac{1000}{P} \right)^{R/C_p} \tag{4.3}$$

where T is the air temperature, $R/C_p = 0.286$ for dry air and P is the barometric pressure.

Next, wind profiles in each of the four height ranges shown in the first four lines, in Table 4.1 were grouped in decreasing order with respect to Ri and divided into nine subgroups in such a way that there were approximately equal numbers of wind profiles in each subgroup. The average value of p and the average value of Ri were then calculated for each subgroup. Graphs of p : Ri were constructed for each height range and are given in Figs. 1-4. The discussion of the graphs will be given in Chapter VI.

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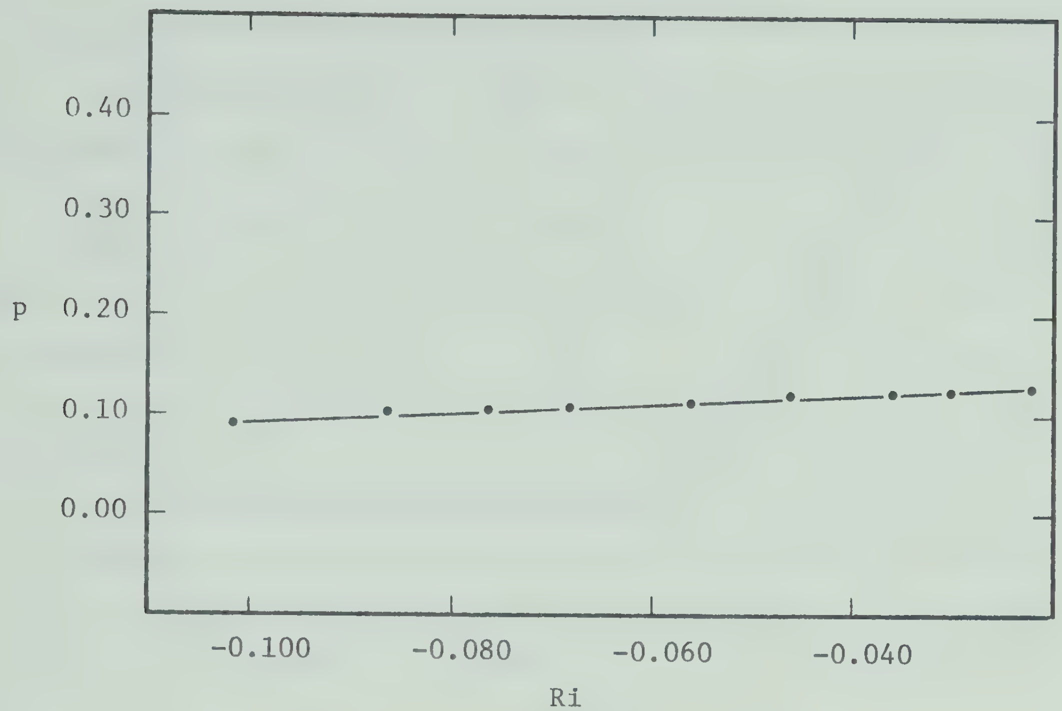

FIG. 1. Variation of the power-law index p with Richardson number in the 0.5 - 16 m layer (Kerang data).

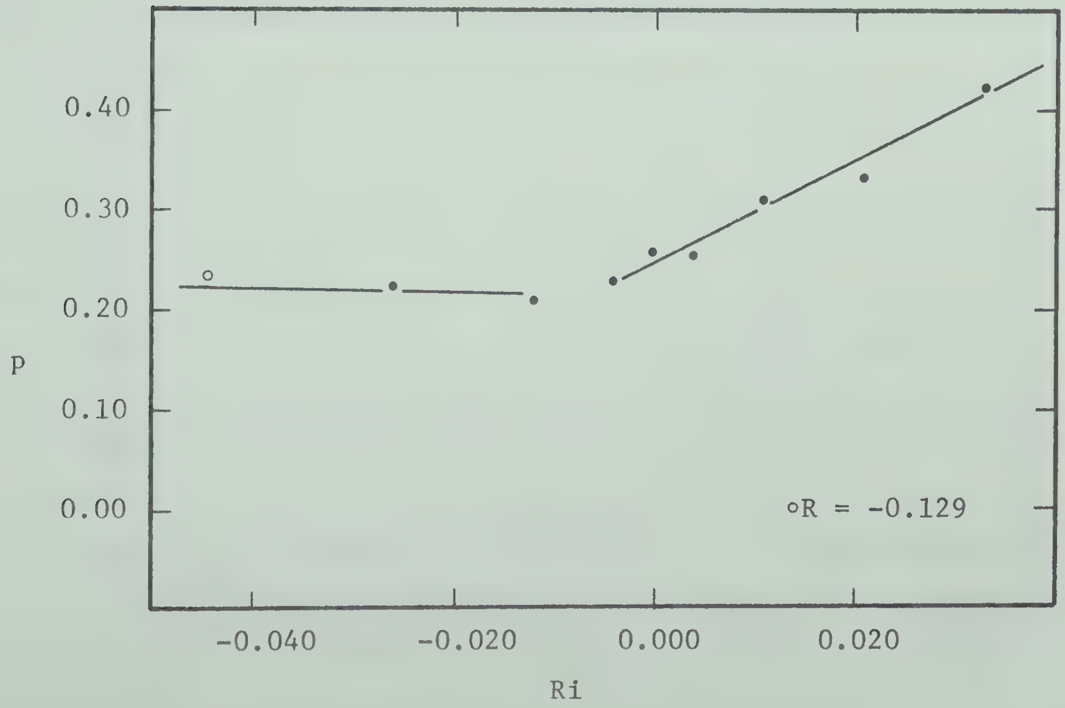

FIG. 2. Variation of the power-law index p with Richardson number in the 0.5 - 16 m layer (Suffield data).

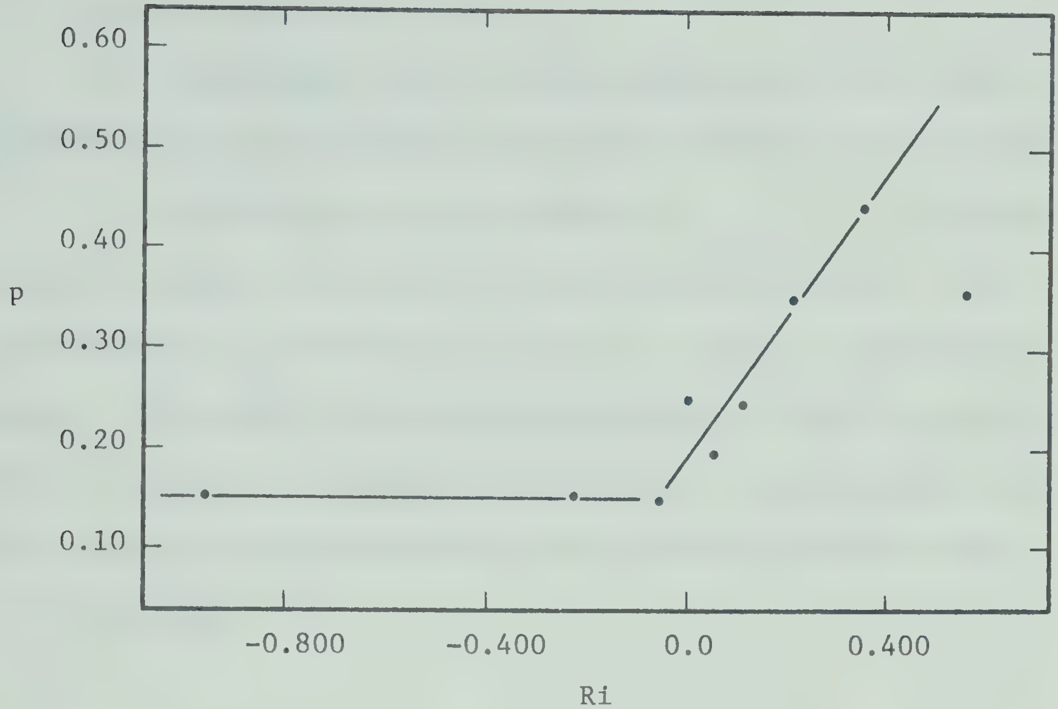

FIG. 3. Variation of the power-law index p with Richardson number in the 2 - 92 m layer (Suffield data).

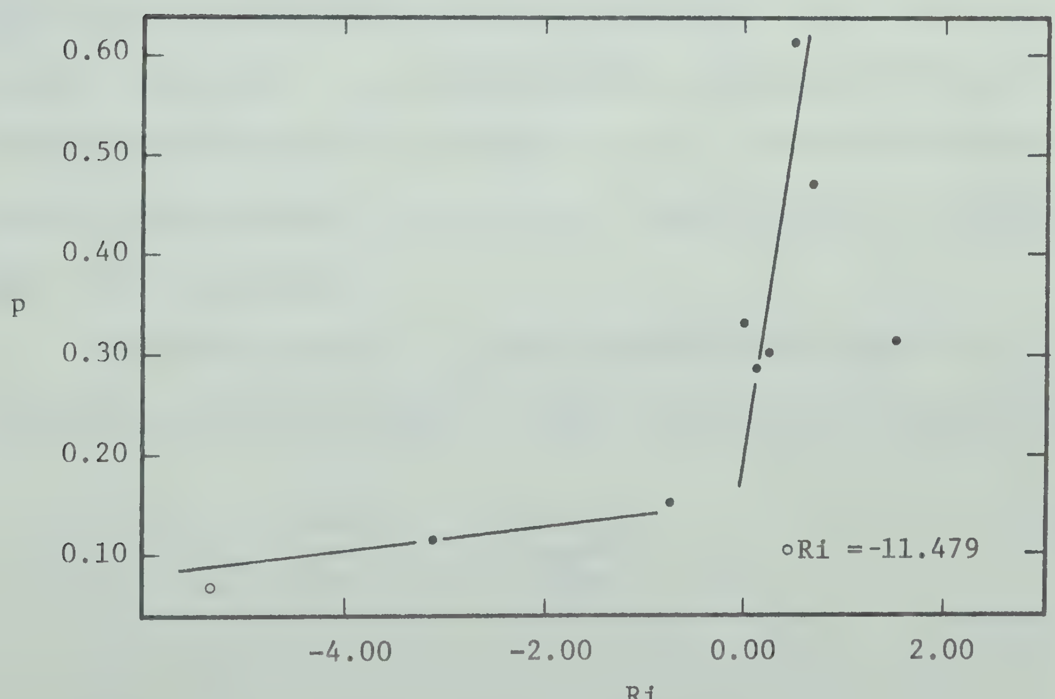

FIG. 4. Variation of the power-law index p with Richardson number in the 16 - 92 m layer (Suffield data).

4.2 The logarithmic Law

The logarithmic law in the form given by Eq. (3.8) was considered as appropriate for the sites at Suffield and at Kerang.

For the Kerang data the parameters u_* and z_o appearing in the logarithmic law were evaluated by Swinbank (1964). Hence it was necessary to evaluate u_* and z_o only for the Suffield terrain. The method of evaluation was similar to that outlined by Panofsky (1963). By taking the derivative of the logarithmic law in Eq. (3.8) it can be shown that in the finite difference form u_* is given as:

$$u_* = k \frac{\Delta U}{\Delta \ln z} \tag{4.4}$$

Furthermore, by applying the boundary condition $U = 0$ at $z = z_o$ it can be shown that z_o is given as the z-intercept of the regression line obtained by plotting $\ln z : U$. Graphs of Suffield wind profiles from which u_* and z_o were calculated are shown in Figs. 25-30, Appendix A.

Once the values of z_o were obtained from graphs the values of u_* were recalculated using the drag coefficient equation:

$$C_z = \frac{u_*}{U_z} = \frac{k}{\ln (z/z_o)} \tag{4.5}$$

The value of von Karman's constant k was assumed to be 0.4 which is the presently accepted value. The values of u_* and z_o calculated from the Suffield wind profiles under neutral or near neutral conditions are given in Table 4.2.

Table 4.2

Calculated values of u_* and z_o for Suffield

Date of profile	T_4-$T_{0.5}$ (F)	$U_{0.5}$ (m sec^{-1})	z_o (cm)	u_* (m sec^{-1}) (graphical)	u_* (m sec^{-1}) (drag coeff.)
12/2/60	-0.4	4.1	1.9	0.48	0.56
13/1/64	-0.3	1.8	3.3	0.25	0.25
23/7/63	-0.3	2.6	3.4	0.38	0.36
1/2/60	-0.3	2.8	2.8	0.35	0.38
14/10/65	0.0	2.8	5.6	0.48	0.38
8/2/60	0.0	3.4	2.9	0.45	0.46

The exact nature of the surface at the times when the profiles were obtained was not available. However, the date of each profile gives some indication of the surface roughness, discussed in Chapter I. It can be observed from Table 4.2 that there is considerable scatter in the calculated values of u_* and z_o. By assigning more weight to the profiles for which the correlation coefficient between $\ln z$ and U was higher the average values of u_* and z_o for Suffield terrain were found to be 0.34 m sec^{-1} and 3 cm respectively.

It was pointed out by Lettau and Davidson (1957) that a vegetation cover of varying density can raise the reference level at which the boundary condition $U = 0$ is satisfied significantly above the average physical soil surface. Under such surface conditions z_o can be determined more accurately by using Eq. (3.9) in the form:

$$U = \frac{u_*}{k} \ln \left(\frac{z + D}{z_o}\right) \tag{4.6}$$

where $D = d+z_o$ is the zero point displacement parameter. The boundary condition now becomes $U = 0$ for $z = -d$. It was shown by Robinson (1962) that, given at least three observations of U and z, Eq. (4.6) can be solved for u_*, z_o, and D using a secant iteration method. A computer program prepared by Robinson was obtained from the Department of Meteorology University of Wisconsin, and values of u_*, z_o, and D were calculated using the Suffield data. The results of these calculations are presented in Table 4.3.

Comparison between Tables 4.2 and 4.3 shows that, in general, the values of u_* are in agreement with one another. On the other hand, the values of z_o in Table 4.3 are, approximately, only half as large as those in Table 4.2. The values of u_*, z_o, and D obtained from the wind profile dated 14/10/65 are somewhat puzzling, particularly the positive D value. Such anomalous D values were also reported by Lettau and Davidson (1957) when they analysed the

Great Plains Turbulence Field Program Data. By omitting the anomalous wind profile from Table 4.3 the average values of u_*, z_o, and D were found to be 0.34 m sec^{-1}, 1.5 cm and -3.2 cm respectively.

Table 4.3

Calculated values of u_*, z_o, and D for Suffield

Date of profile	$T_4-T_{0.5}$ (F)	u_* (m sec^{-1})	z_o (cm)	D (cm)
12/2/60	-0.4	0.44	1.0	-3.7
13/1/64	-0.3	0.22	1.7	-1.2
23/7/63	-0.3	0.35	1.9	-5.4
1/2/60	-0.3	0.31	1.7	-2.3
14/10/65	0.0	0.65	19.4	64.5
8/2/60	0.0	0.40	1.5	-3.5

4.3 The log-Linear Law

a. The Monin-Obukhov log-linear law

The Monin-Obukhov log-linear law given by Eq. (3.26) was considered in the analyses. Because heat flux measurements were not available the gradient length L', given by Eq. (3.23), was calculated for each profile. The Monin-Obukhov constant α was calculated from the wind profiles in each height range by plotting the quantity $\ln z + \phi(z/L')$ as a function of wind speed and using the least squares method. It was found that the value of α was different for free

convection, forced convection, and inversion conditions. Moreover, the value of α was different in different layers. The calculated values of α for each height range are given in Table 4.4.

Table 4.4

Calculated values of α

Height range		α	Range of Ri
Suffield	2 - 92 m	0.058	$Ri \leqslant -0.500$
		0.58	$-0.500 < Ri < 0$
		4.5	$0 < Ri < 0.140$
		1.0	$Ri \geqslant 0.140$
Suffield	16 - 92 m	0.058	$Ri < 0$
		1.0	$0 < Ri \leqslant 0.390$
		0.058	$Ri > 0.390$
Suffield	0.5-16 m	0.058	$Ri < 0$
		6.0	$Ri > 0$
Kerang	0.5-16 m	0.58	$Ri < 0$

The values of α in Table 4.4 bear some resemblance to those obtained by Taylor (1960) for forced convection, free convection, and inversion regimes. Moreover, the value of α obtained from Kerang data is in agreement with the value $\alpha = 0.6$ used by Bernstein (1966) when he used the Kerang data to compare different forms of the log-linear law.

b. The Brooks log-power law

Following the Brooks et al (1963) suggestion that a better fit to observed wind profiles was possible by using a square root modification of the log-linear law, the Suffield and Kerang data were fitted to Eq. (3.36). As in the case of the Monin-Obukhov log-linear law the gradient length L' was used in the analyses. The parameter γ' which appears in the Brooks log-power law was calculated from the wind profiles in each height range using the least squares method. The calculated values of γ' are given in Table 4.5.

Table 4.5

Calculated values of γ'

Height range	γ'	Range of Ri
Suffield 2 - 92 m	1.0 0.1	Ri > 0 Ri < 0
Suffield 16 - 92 m	1.5 0.1	Ri > 0 Ri < 0
Suffield 0.5-16 m	1.5 0.1	Ri > 0 Ri < 0
Kerang 0.5-15 m	1.0	Ri < 0

A study of Table 4.5 reveals that the parameter γ' plays a similar role in the Brooks log-power law as the parameter α does in the Monin-Obukhov log-linear law. However, it appears that γ' does not vary with the thickness of the layer considered.

c. The modified log-linear law

After some experimentation with different wind profile models discussed in Chapter III it was found that the data were represented well by a modified log-linear law in the form:

$$U = \frac{u_*}{k} \left[\ln \frac{z}{z_o} + \left(\frac{1}{L_o}\right)^{\frac{1}{2}} \frac{z}{\sqrt{L}} \right] \tag{4.7}$$

Here L_o is a constant length determined from the wind profile data and L is the gradient length given by Eq. (3.23). Calculations showed that L_o is relatively small for $Ri > 0$ and large for $Ri < 0$. Furthermore, L_o is different in each height range. The calculated values of L_o for each height range are given in Table 4.6.

Table 4.6

Calculated values of L_o

Height range	L_o (m)	Range of Ri
Suffield 2 - 92 m	8.2 8.2×10^2	$Ri > 0$ $Ri < 0$
Suffield 16 - 92 m	25 25×10^2	$Ri > 0$ $Ri < 0$
Suffield 0.5-16 m	0.07 15.6×10^3	$Ri > 0$ $Ri < 0$
Kerang 0.5-16 m	30	$Ri < 0$

It may be observed from Table 4.6 that there is a relatively large difference in L_o between stable and unstable conditions. However, it should be noted that in Eq. (4.7) the numerical value of $(1/L_o)^{\frac{1}{2}}$ is of importance and not L_o itself.

The question now remains which of the wind profile laws fits the data most accurately. This question will be discussed in the next chapter.

CHAPTER V

COMPARISON BETWEEN THE DIFFERENT WIND PROFILE MODELS

One of the fundamental questions in micrometeorology which still awaits the answer is to determine which of the wind profile models describes the actual conditions best. Relatively few comparative studies between different wind models were done in the past. The method of comparison, generally, varied with each investigator and until the present time there is no standard way of making the comparison.

Frost (1948) compared the power law with the logarithmic law. The results showed that over open grassland, for heights below 10 m, the power law representation was better than that given by the logarithmic law. The wind-speed values given by the logarithmic law in the layer below 10 m were too low. In the 10-100 m layer the wind speeds were represented equally well by the two laws. On the other hand, in the 100-800 m layer the logarithmic law was better while the power law gave too high speeds. When the roughness of the surface was taken into account, it was found that for relatively smooth surfaces, with $z_0 \leq 1$ cm, the wind profiles can be best represented, up to several hundred meters, by a logarithmic law. However, for very rough surfaces the power law gave a better representation.

Johnson (1959) analysed wind profiles extending from 0.5-16 m. The measurements were taken over prairie grass and over snow surface. It was found that under all stability conditions and for each surface

roughness the power law gave equal, and in most cases better, results than the logarithmic law. Furthermore, the results indicated that under all stability conditions the wind speed increased with height more rapidly than predicted by the logarithmic Law.

DeMarrais (1959) considered the power law in the form:

$$U = U_1 \left(\frac{z}{z_1}\right)^p \tag{5.1}$$

He then used the observed wind speed at 11 m and the value of p for each ten-minute period to compute the wind speed at 46 m. Next, he computed the 46 m wind speed using the logarithmic law in the form:

$$U_2 = U_3 - (U_3 - U_1) \frac{\log(z_3/z_2)}{\log(z_3/z_1)} \tag{5.2}$$

where the subscripts 1, 2, and 3 refer to 11, 46 and 125 m respectively. The computed 46-m wind speeds, for each law, were then compared with the observed wind speeds and the mean error for each temperature lapse-rate class was computed. The results showed that the logarithmic law had greater error than the power law for all except the superadiabatic lapse rates.

Volkovitskaya and Mashkova (1963) analysed the wind data obtained from the Russian 300-m tower. They found that in the bottom 25-30 m layer the wind speed varies according to a logarithmic law while in the overlying 25-300 m layer the wind profile deviates from

the logarithmic law and can be approximated by a power law.

Thuillier and Lappe (1964) analysed the data obtained from the 1420-ft Cedar Hill tower located near Dallas, Texas. They found that for near-adiabatic and slightly unstable conditions the logarithmic law represents the data well up to a height of 300-400 ft. Above this height the wind speed was found to be nearly constant with height. For stable non-inversion conditions the power law was found to fit the data better than the logarithmic law while under inversion conditions the wind profiles were highly variable and dependent on the nature of the inversion present.

The main area of research during the recent years involved the comparisons of different forms of the log-linear law. Panofsky, Blackadar and McVehil (1960) plotted the non-dimensional shear S versus $\ln \gamma z/L$, where γ was assumed to be a constant, and compared the theoretical relationship between S and $\gamma z/L$, for KEYPS and Monin-Obukhov functions, with observed values obtained at five different locations. The comparison showed that the KEYPS function fits the observations in unstable air better than the Monin-Obukhov function. However, on the stable side neither function fits the observed data well.

Panofsky (1963) made a theoretical comparison between the Monin-Obukhov, Webb, and KEYPS profiles by plotting the universal function $\ln \psi : \ln z/L$. The comparison showed that for small values of $\ln z/L$ the three profiles are identical. However, for

ln $z/L > 0.1$ the KEYPS and Webb profiles agree well with each other while the Monin-Obukhov profile diverges considerably.

Bernstein (1966) used the data obtained at O'Neill, Nebraska, and at Kerang, Australia, and compared the Monin-Obukhov, KEYPS, and Swinbank profiles with actual observations. The comparison was made by plotting the logarithm of the non-dimensional wind shear S against the logarithm of $-z/L$ under unstable conditions. He found that the data were not of sufficient accuracy to distinguish among the different hypotheses and concluded that the presently available data are not sufficient to verify or refute any of the three models.

Hansen (1966) compared nine wind-profile models for the diabatic boundary layer (including Monin-Obukhov, Brooks, KEYPS, Webb, Businger, and Swinbank models). The data used in the analyses were from O'Neill, Nebraska, Kerang, Australia, and White Sands Missile Range, New Mexico. The method of comparison was similar to that of Bernstein (1966). Hansen found that in the forced convection regime, $0 > Ri > -0.02$, all models allowed accurate determination of the roughness length and would give good estimates of the stress and vertical heat flux. However, in the transition zone between forced and free convection none of the models gave good results beyond the junction height $Ri = -0.0317$. Furthermore, in the free convection regime, $Ri < -0.05$, none of the models fitted the experimental data. For the White Sands data, which was obtained under non-stationary conditions owing to the non-uniform terrain and the formation of the internal momentum boundaries caused by roughness discontinuities,

none of the models was found to be valid. Consequently, Hansen concluded that the present surface boundary layer models have limited use in less than ideal situations.

The comparisons between the power law and the log-linear law are rare. The main difficulty is that the former law is empirical while the latter is theoretical. This results in the absence of common factors, making a direct comparison difficult.

The method of comparison adopted in this study attempts to determine the accuracy with which the least-squares profiles represent the observed data. This method is somewhat similar to that used by McVehil (1964). For each of the four height ranges shown in Table 4.4 the wind profiles were grouped in decreasing order according to the gradient Richardson number into eight or nine subgroups, depending on the number of samples. The grouping was done in such a way that there were approximately equal numbers of samples in each subgroup. The average value of Ri was then obtained for each subgroup. Next, for each subgroup the correlation coefficients, discussed in the previous chapter, were transformed into Fisher's z' - statistic and weighted before being combined. The equations used in transforming the correlation coefficients are given in Appendix B. A procedure similar to that outlined by Brooks and Carruthers (1953) was used to apply the weights. This procedure was repeated for each wind-profile law. The results of comparison between the power law, the logarithmic law, and the Monin-Obukhov log-linear law are summarized in Figs. 5-8. Similar method of comparison was used to compare the three log-linear

wind-profile models discussed in Chapter IV, section 3. The results from this comparison are displayed graphically in Figs. 9-12. These graphs will be discussed further in the next chapter.

The next question of importance in the analyses was to determine if the differences between different wind profile laws were significant. The five per cent level confidence intervals were considered appropriate to determine the significance. The significance tests were performed in two parts.

First, for each subgroup, in addition to the weighted correlation coefficient, the five per cent level confidence intervals were calculated. The equations used in calculations are given in Appendix B. Figs. 13-16 show the comparison between the five per cent level confidence intervals for power law, logarithmic law, and Monin-Obukhov log-linear law. In addition, the comparison between the confidence intervals for Brooks log-power law, modified log-linear law, and Monin-Obukhov log-linear law is given in Figs. 17-20.

Second, instead of computing the weighted correlation coefficients and the corresponding confidence intervals for each subgroup, an average weighted correlation coefficient together with the five per cent level confidence intervals was calculated for each of the four sets of data. The comparison of significant differences between power law, logarithmic law, and Monin-Obukhov log-linear law, for each set of data, is displayed graphically in Figs. 21-24 while the comparison between Brooks log-power law, modified log-linear law, and Monin-Obukhov log-linear law is given in Table 5.1.

Table 5.1

Comparison of significant differences between Brooks log-power law, modified log-linear law, and Monin-Obukhov log-linear law

Wind profile law	Height range (m)	No. of obs.	Average weighted correlation coefficient	5% level confidence intervals
Brooks log-power law	Suffield 2-92	89	0.985	0.982-0.988
	Suffield 16-92	78	0.985	0.981-0.989
	Suffield 0.5-16	34	0.990	0.985-0.994
	Kerang 0.5-16	34	0.999	0.998-0.999
Modified log-linear law	Suffield 2-92	89	0.988	0.985-0.990
	Suffield 16-92	78	0.985	0.981-0.989
	Suffield 0.5-16	34	0.996	0.994-0.998
	Kerang 0.5-16	34	0.999	0.998-0.999
Monin-Obukhov log-linear law	Suffield 2-92	89	0.987	0.984-0.989
	Suffield 16-92	78	0.987	0.982-0.990
	Suffield 0.5-16	34	0.993	0.989-0.995
	Kerang 0.5-16	34	0.999	0.998-0.999

This concludes the analyses of the wind profile observations, and in the following chapter the assessment of the results, here presented, will be made.

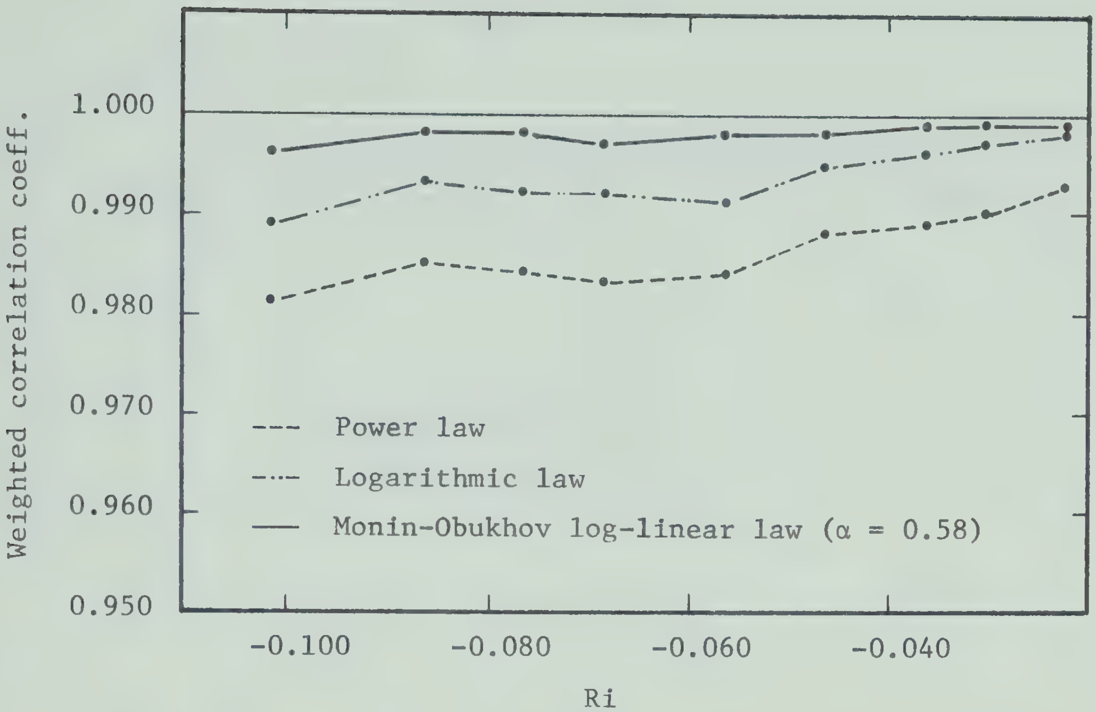

FIG. 5. Comparison between Monin-Obukhov log-linear law, logarithmic law, and power law in the 0.5 - 16 m layer (Kerang data).

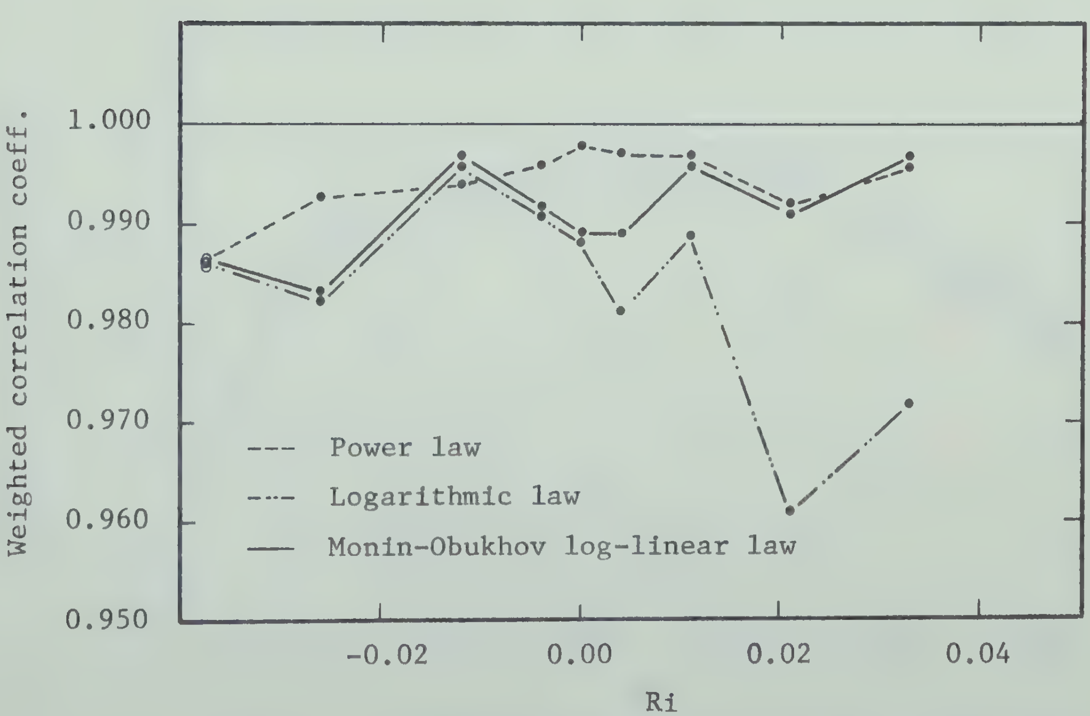

FIG. 6. Comparison between Monin-Obukhov log-linear law, logarithmic law, and power law in the 0.5 - 16 m layer (Suffield data). °Ri =-0.129

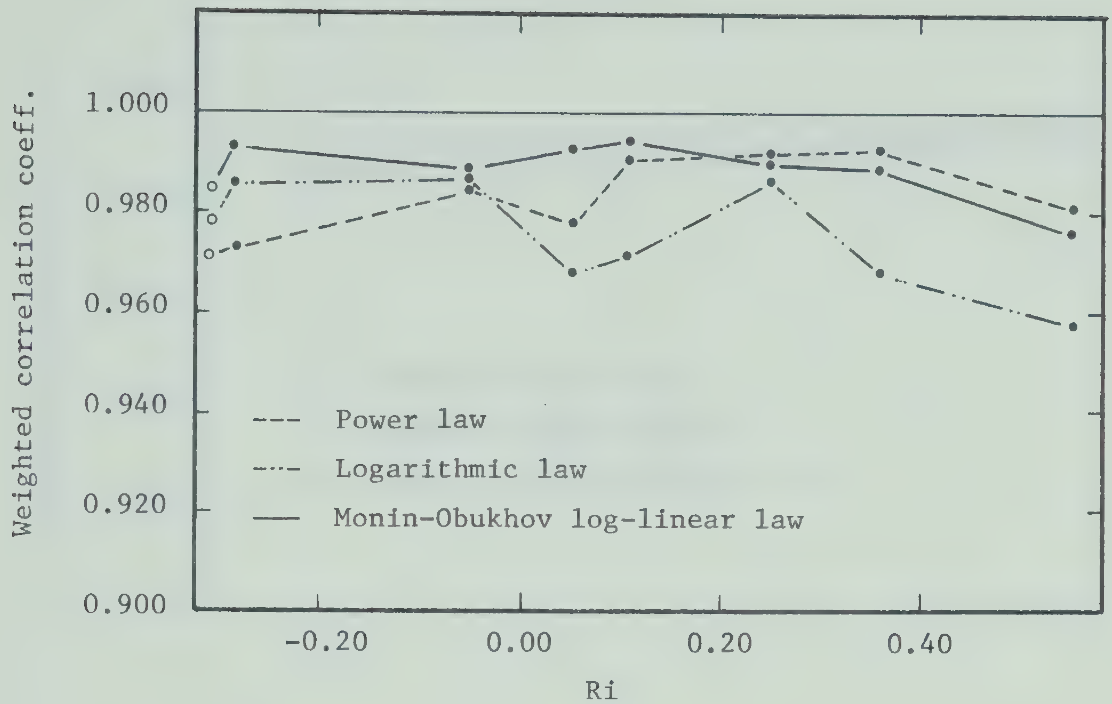

FIG. 7. Comparison between Monin-Obukhov log-linear law, logarithmic law, and power law in the 2 - 92 m layer (Suffield data). °Ri = -0.960

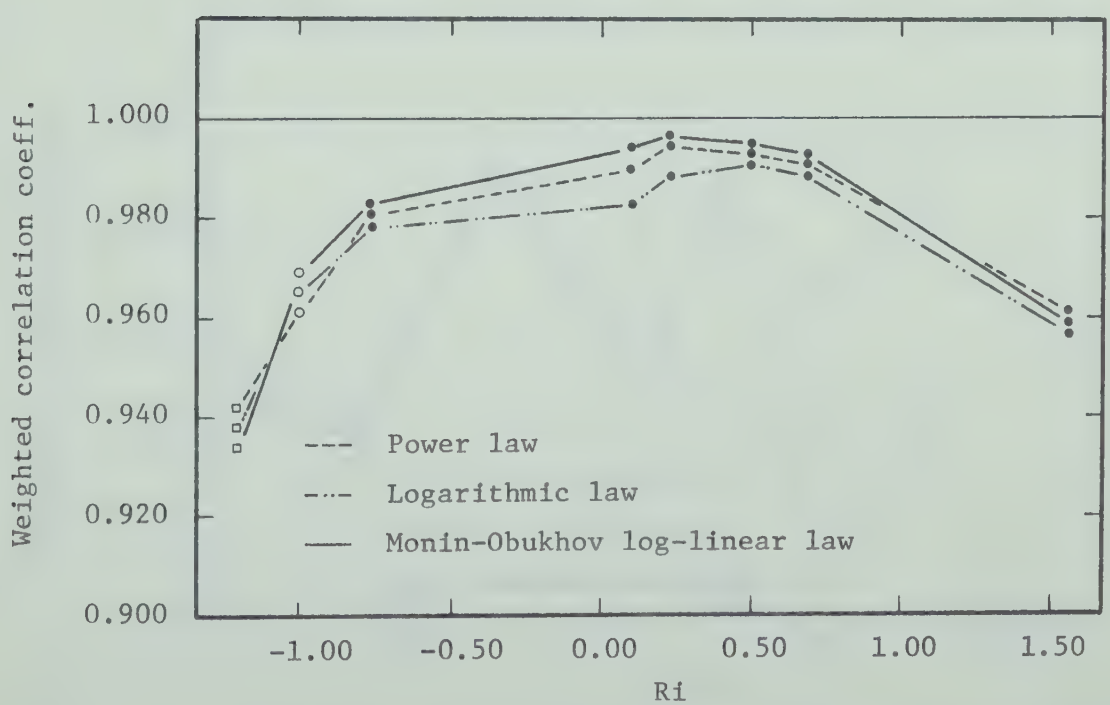

FIG. 8. Comparison between Monin-Obukhov log-linear law, logarithmic law, and power law in the 16 - 92 m layer (Suffield data). °Ri = -3.084, ▫Ri = -11.479

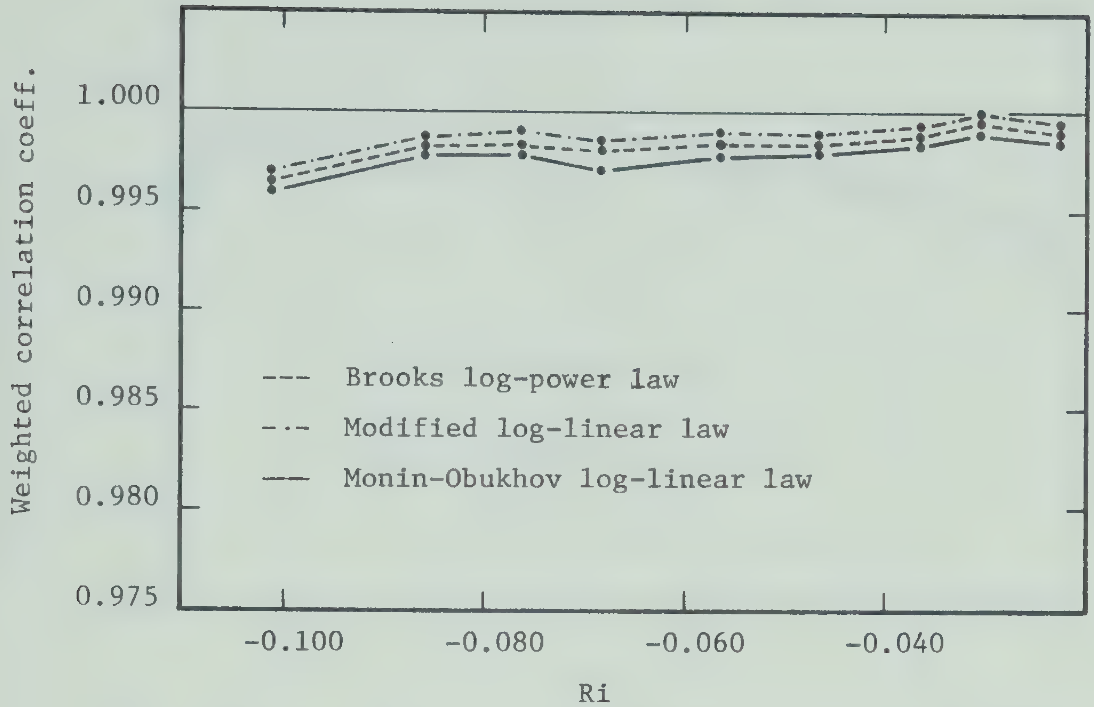

FIG. 9. Comparison between Monin-Obukhov log-linear law, Brooks log-power law, and modified log-linear law in the 0.5 - 16 m layer (Kerang data).

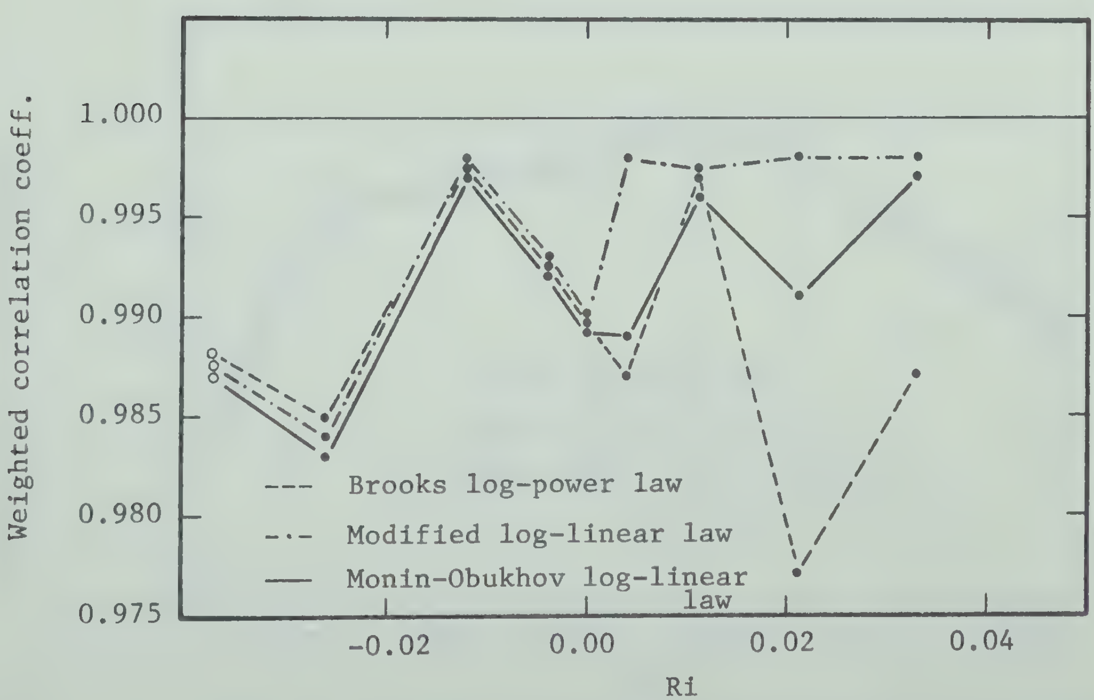

FIG. 10. Comparison between Monin-Obukhov log-linear law, Brooks log-power law, and modified log-linear law in the 0.5 - 16 m layer (Suffield data). °Ri =-0.129

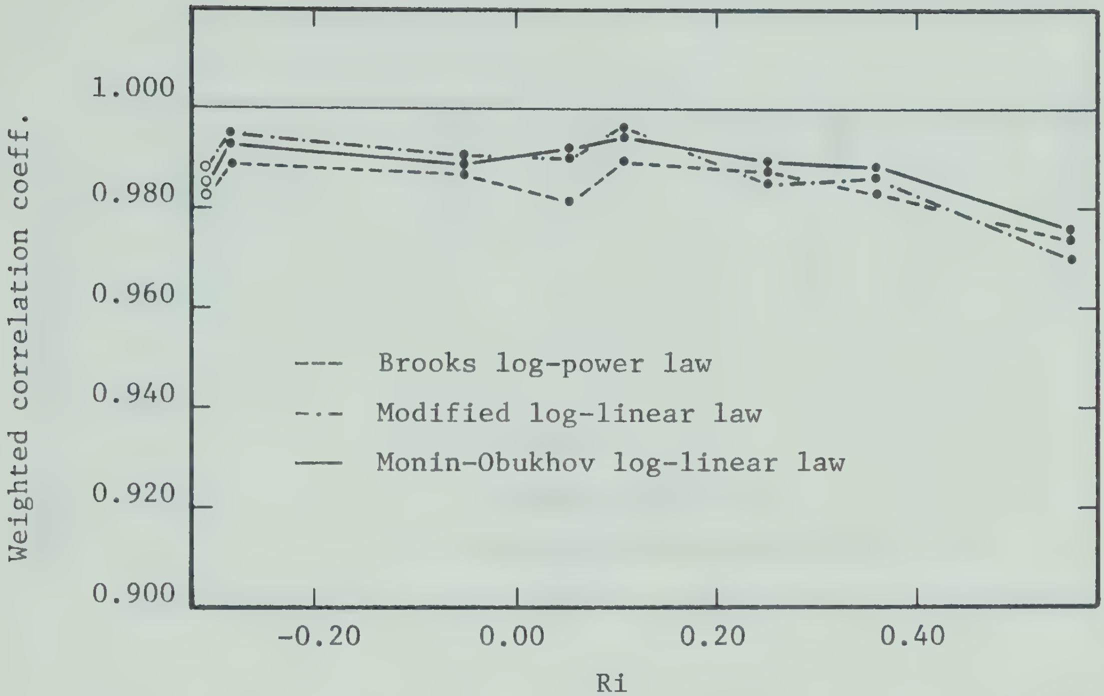

FIG. 11. Comparison between Monin-Obukhov log-linear law, Brooks log-power law, and modified log-linear law in the 2 - 92 m layer (Suffield data). °Ri = -0.960

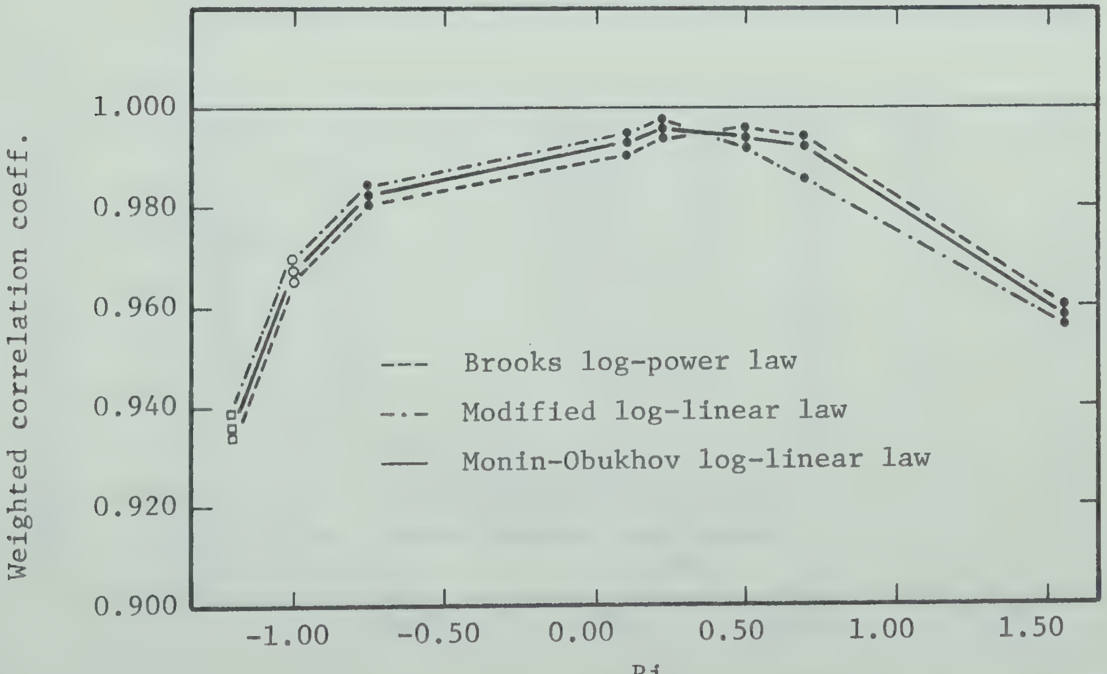

FIG. 12. Comparison between Monin-Obukhov log-linear law, Brooks log-power law, and modified log-linear law in the 16 - 92 m layer (Suffield data). °Ri =-3.084, ▫Ri = -11.479

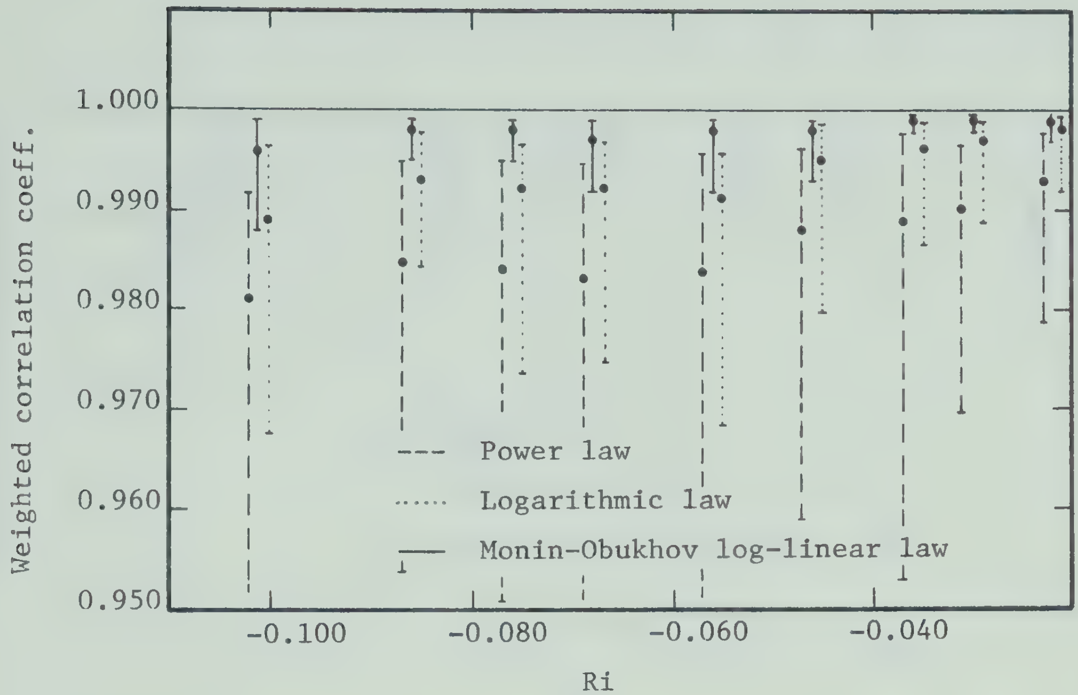

FIG. 13. Comparison between the five per cent level confidence intervals for different wind-profile laws in the 0.5 - 16 m layer (Kerang data).

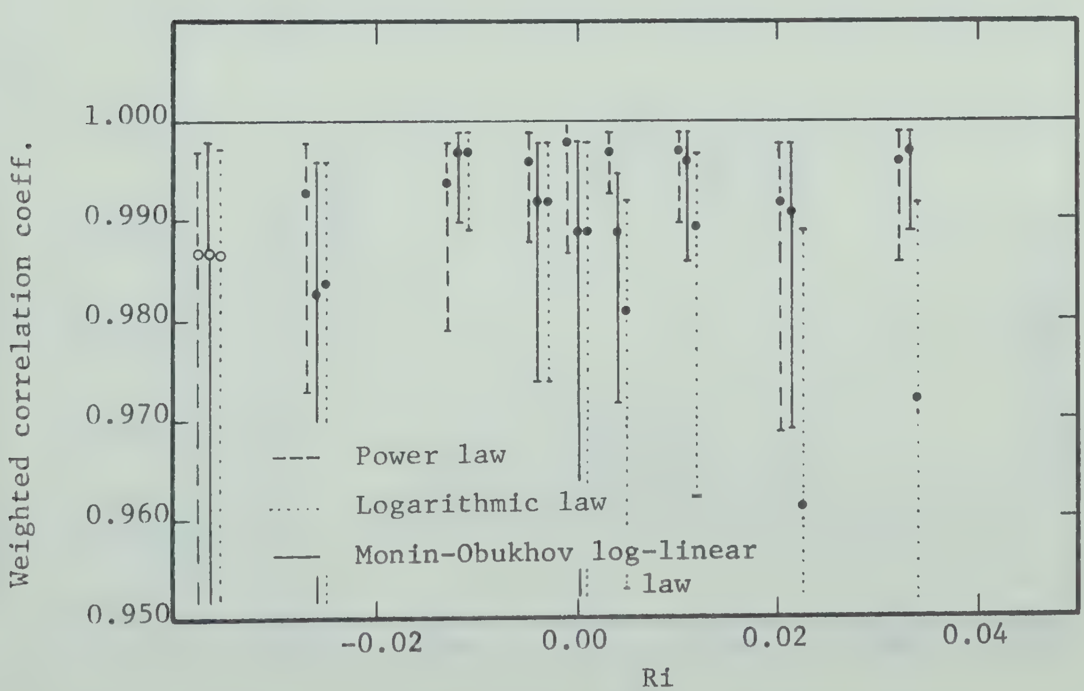

FIG. 14. Comparison between the five per cent level confidence intervals for different wind-profile laws in the 0.5 - 16 m layer (Suffield data). °Ri = -0.129

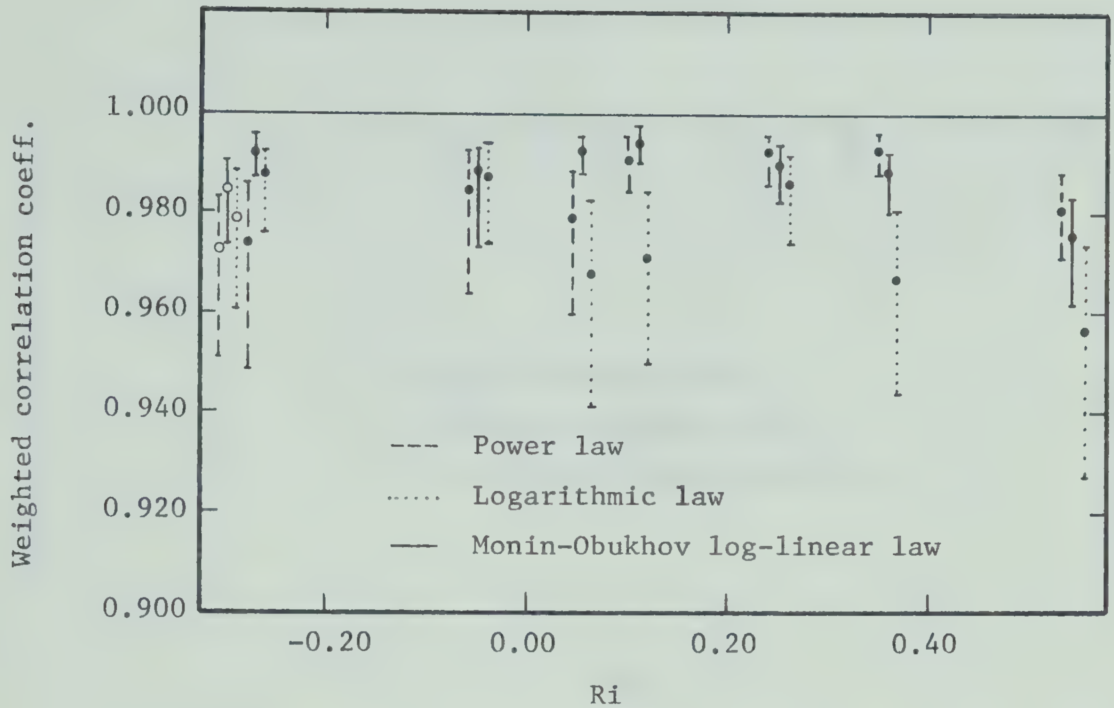

FIG. 15. Comparison between the five per cent level confidence intervals for different wind-profile laws in the 2 - 92 m layer (Suffield data). °Ri = -0.960

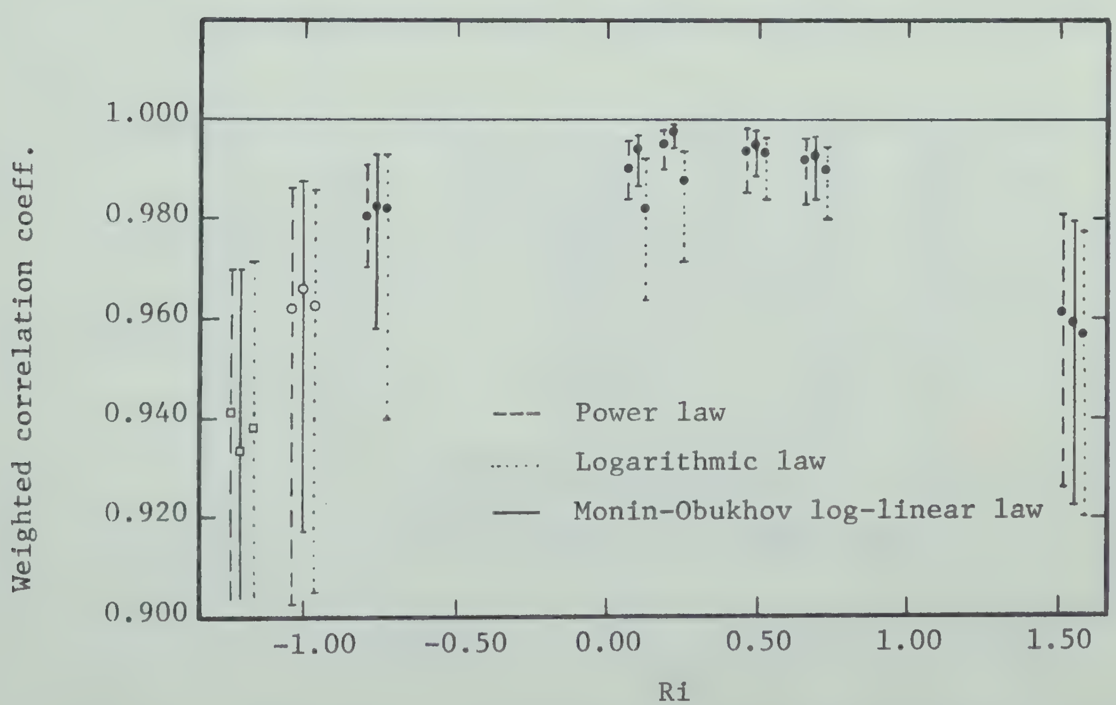

FIG. 16. Comparison between the five per cent level confidence intervals for different wind-profile laws in the 16 - 92 m layer (Suffield data). °Ri = -3.084, ▫Ri = -11.479

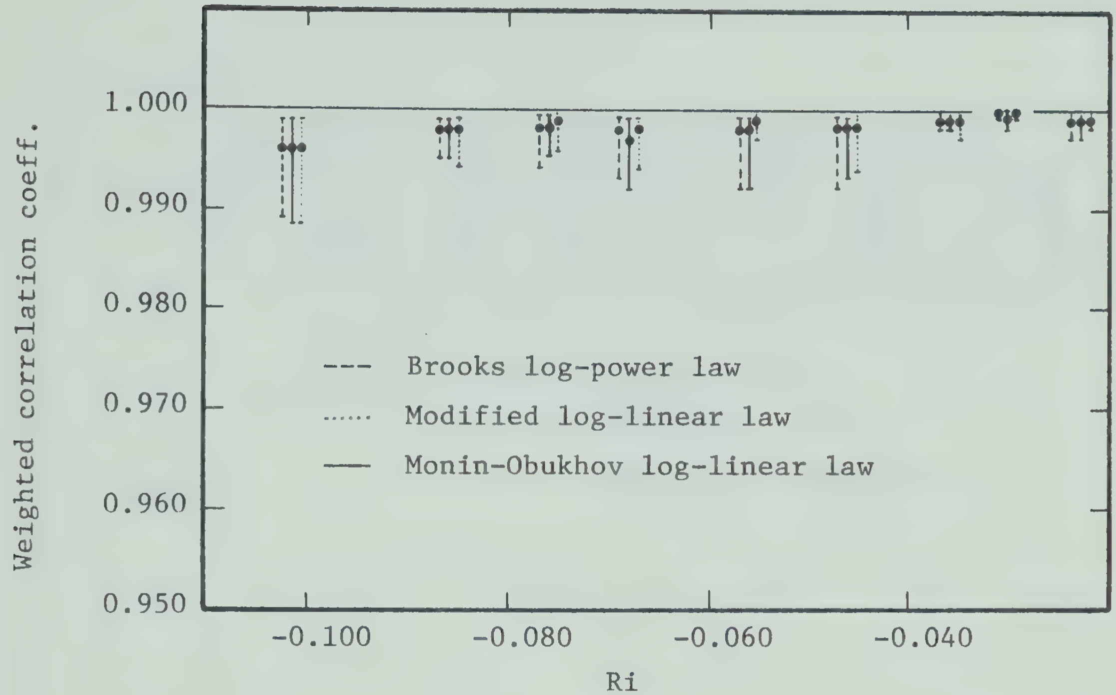

FIG. 17. Comparison between the five per cent level confidence intervals for different wind-profile lows in the 0.5 - 16 m layer (Kerang data).

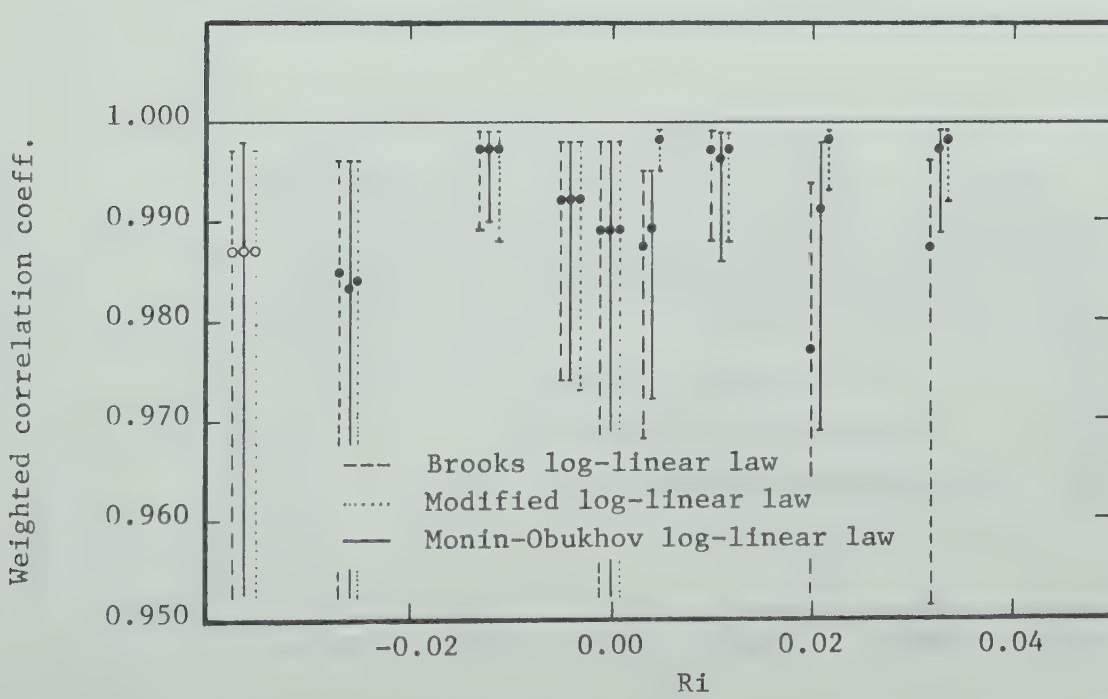

FIG. 18. Comparison between the five per cent level confidence intervals for different wind-profile laws in the 0.5 - 16 m layer (Suffield data). °Ri = -0.129

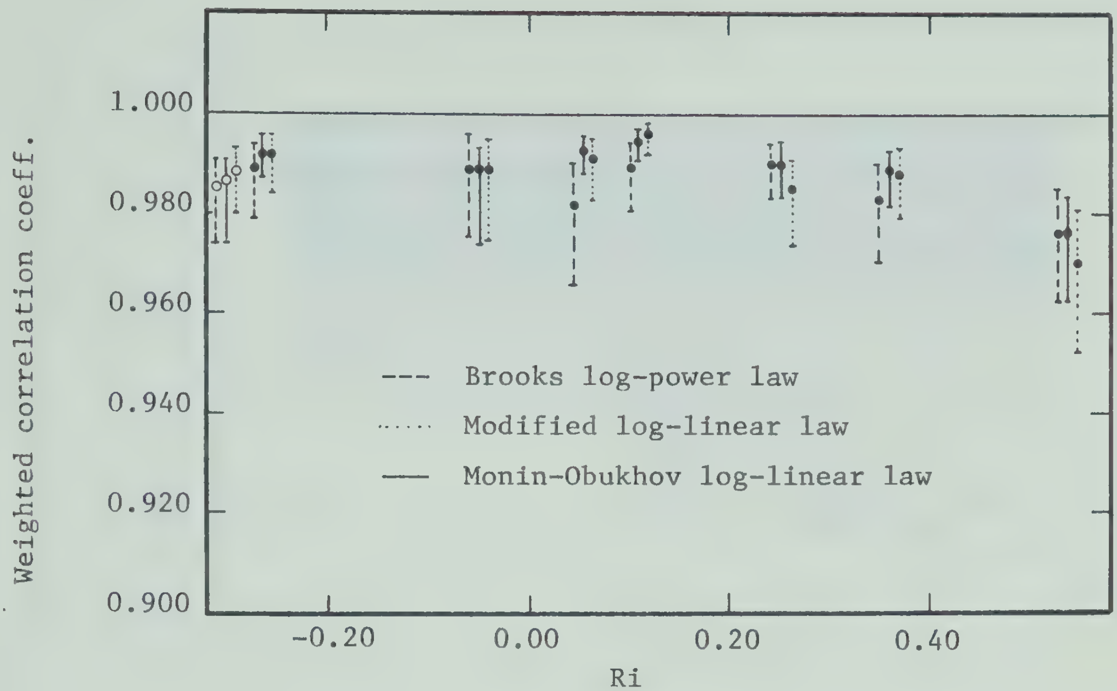

FIG. 19. Comparison between the five per cent level confidence intervals for different wind-profile laws in the 2 - 92 m layer (Suffield data). °Ri = -0.960

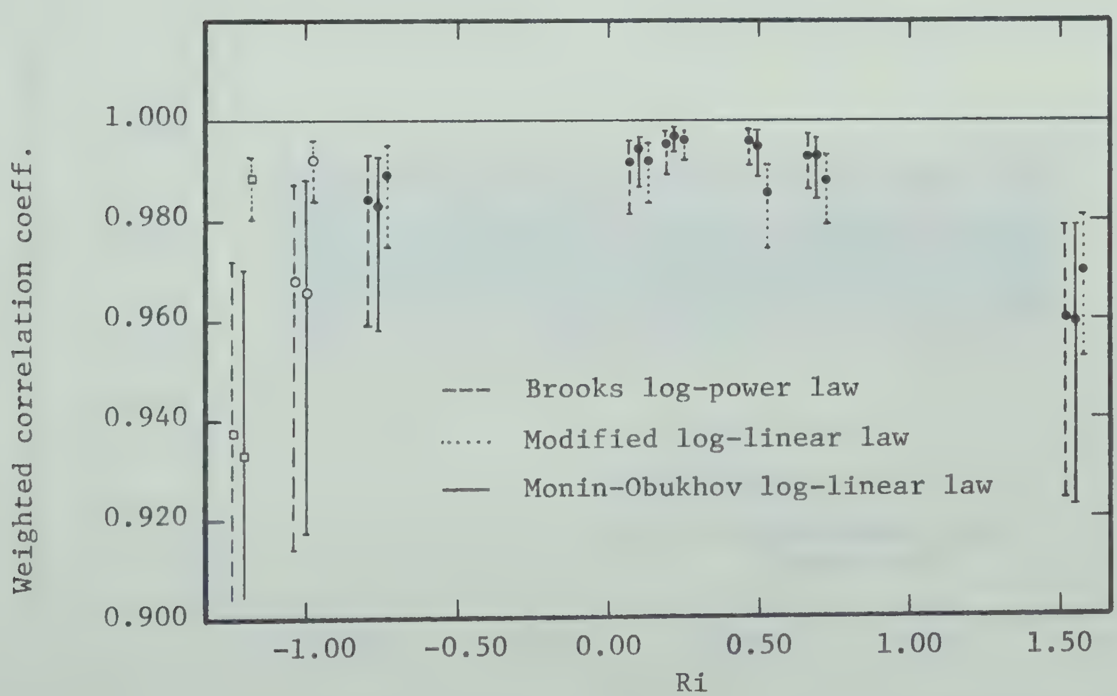

FIG. 20. Comparison between the five per cent level confidence intervals for different wind-profile laws in the 16 - 92 m layer (Suffield data). °Ri = -3.084, ▫Ri = -11.479

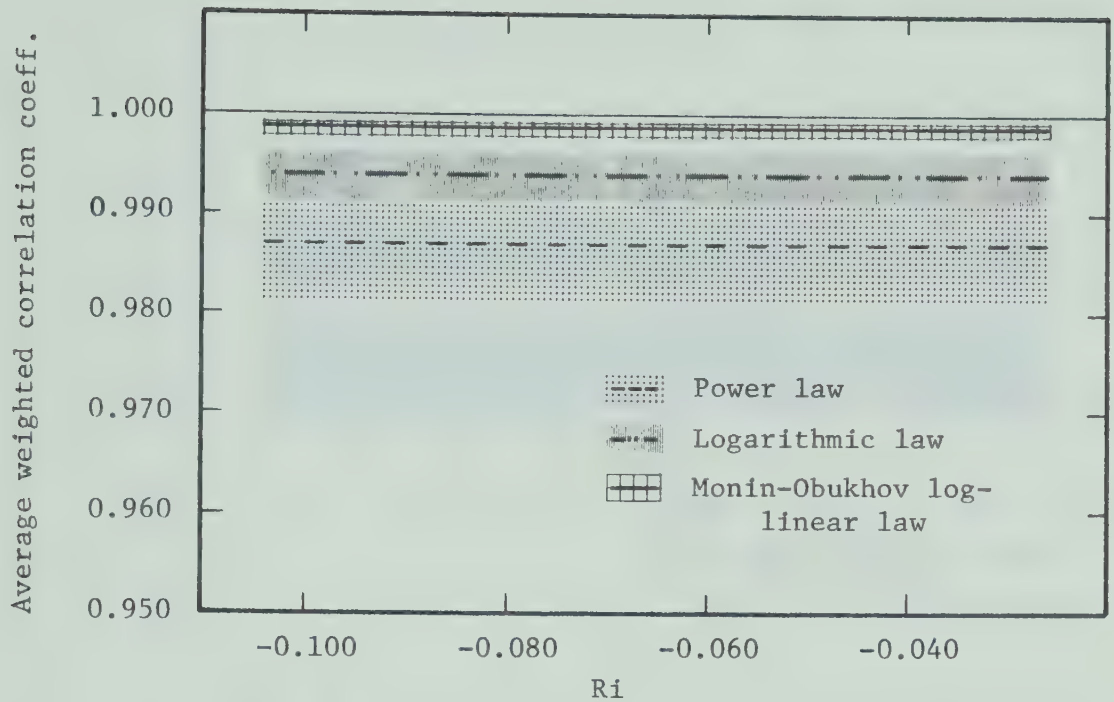

Fig. 21. Diagram to illustrate the significant differences between different wind-profile laws. The shaded areas represent the five per cent confidence intervals for the 0.5 - 16 m layer (Kerang data).

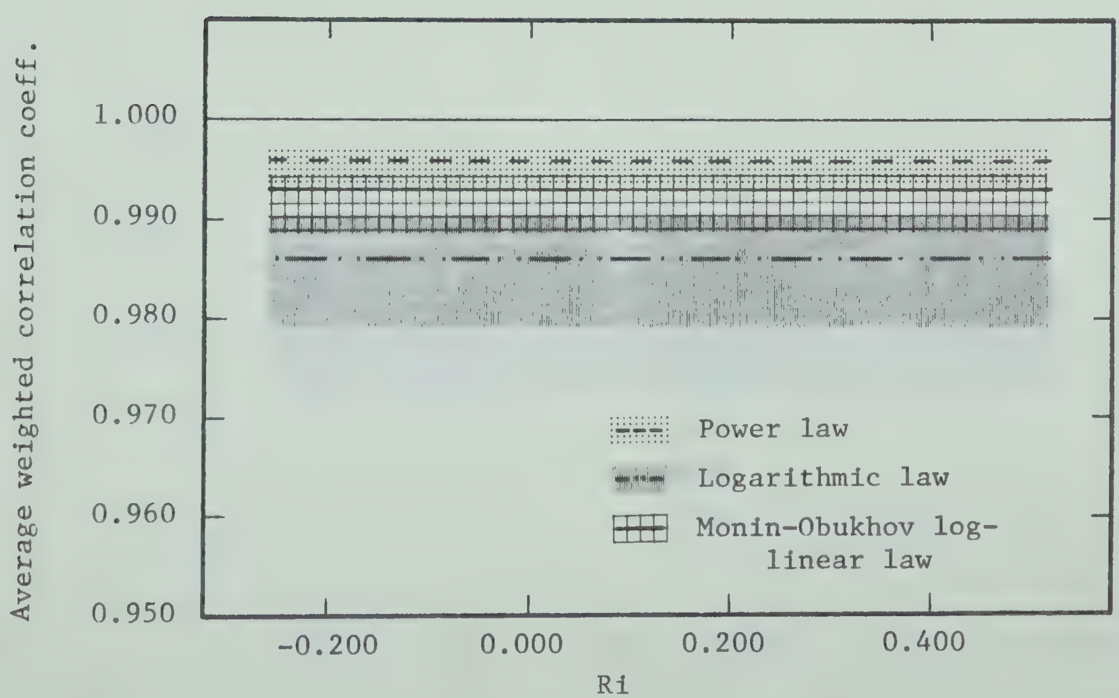

Fig. 22. Diagram to illustrate the significant differences between different wind-profile laws. The shaded areas represent the five per cent confidence intervals for the 0.5 - 16 m layer (Suffield data).

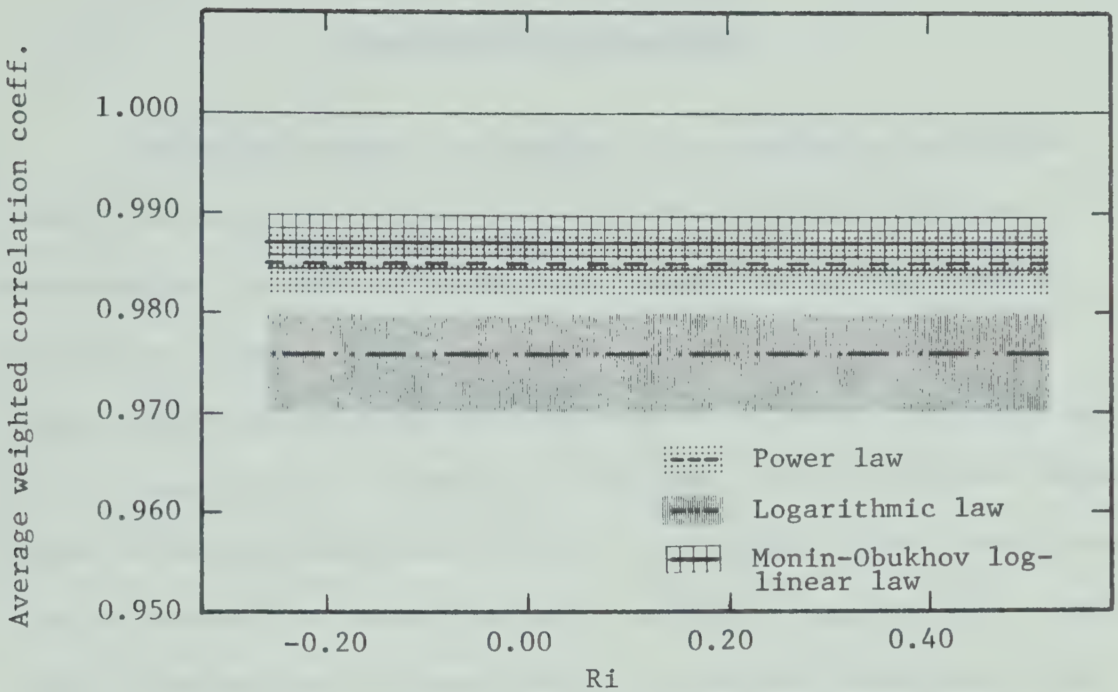

Fig. 23. Diagram to illustrate the significant differences between different wind-profile laws. The shaded areas represent the five per cent confidence intervals for the 2 - 92 m layer (Suffield data).

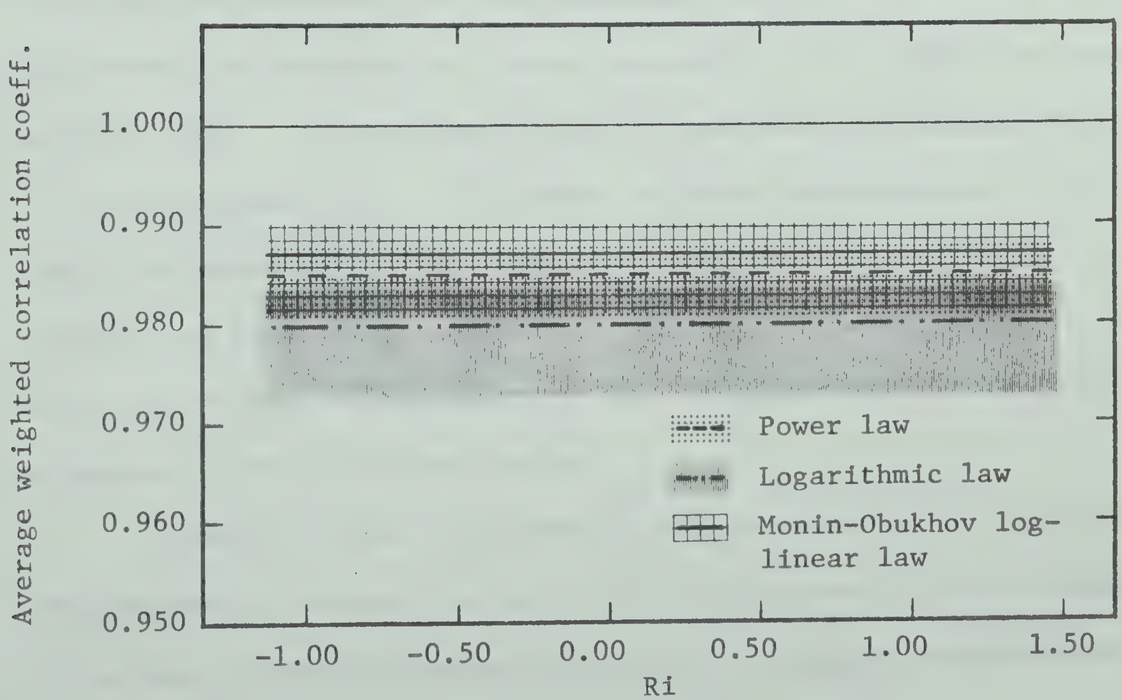

Fig. 24. Diagram to illustrate the significant differences between different wind-profile laws. The shaded areas represent the five per cent confidence intervals for the 16 - 92 m layer (Suffield data).

CHAPTER VI

DISCUSSION OF THE RESULTS

Before discussing the results it is desirable to consider some of the obvious sources of error. The Suffield wind-speed measurements, as well as temperature measurements, were given to one decimal place. It was estimated that at Suffield winds and temperatures were measured with an accuracy of about 5 cm sec^{-1} and 0.05 F respectively. According to Swinbank (1964), winds and temperatures at Kerang, which were given to two decimal places, were measured with an accuracy of about 2 cm sec^{-1} and 0.02 C respectively. Furthermore, as pointed out by Hage (1961), the Suffield wind-speed measurements were made with cup anemometers and are likely to contain systematic errors because of turbulence. In addition, in the early 16-m tower experiments, the top anemometer (16 m) was mounted on a mast above the tower and all lower anemometers were mounted on booms into the wind. Studies showed that the boom-mounted anemometers were too low by amounts up to 15% because of tower interference. It can readily be shown that if the winds and temperatures used in the analyses are in error in opposite senses the consequent error in the calculated values is significant. This is particularly true in calculations of the Richardson numbers.

A study of Figs. 1-4 reveals that the variation of the power law exponent p depends on the state of turbulence in the atmosphere. In the free convection regime the value of p remains approximately

[illegible]

constant while in the forced convection regime p increases rapidly with increasing values of Ri. The comparison of Figs. 1-4 shows that the discontinuity in the $p : Ri$ curve occurs in the vicinity of $Ri = -0.03$, in agreement with Priestley's (1959) value of the free convection-forced convection transition.

Figs. 5-8 show the comparison of the accuracy with which the power law, the logarithmic law, and the log-linear law represent the observed data. If the observed wind speeds fitted a particular law with one-hundred per cent accuracy, the weighted correlation coefficient would be equal to unity. It may be observed from the graphs that the Kerang data are represented most accurately by the Monin-Obukhov log-linear law while the power law is the least valid. For the 0.5-16 m layer at Suffield it is observed that the power law represents the data as well as or better than either of the other two laws with the exception of a short interval in vicinity of $Ri = -0.01$. This finding is in agreement with that found in an independent study by Johnson (1959) when he compared the logarithmic and the power law using the Suffield data. Figs. 7 and 8 show that in the 2-92 m and 16-92 m layers the log-linear law represents the data best in the near-adiabatic vertical temperature stratification while in the regions far from adiabatic the power law gives a better fit. This result is in agreement with the theoretical derivation of the Monin-Obukhov log-linear law which, according to theory, is valid only in the near-adiabatic conditions.

The comparison of accuracy with which the Brooks log-power law, the modified log-linear law, and the Monin-Obukhov log-linear law represent the data is shown in Figs. 9-12. It may be observed that for the Kerang data the differences between the three laws are very small. There exists a systematic tendency for all three laws to fit the data more accurately in vicinity of the adiabatic condition. However, there is no significant departure of any particular law from the remaining two. On the other hand, for the 0.5-16 m layer at Suffield the differences among the three laws are negligible for $Ri < 0$ while for $Ri > 0$ the differences become more pronounced. It appears that the Brooks log-power law becomes less valid under the inversion conditions, whereas the modified log-linear law represents the data well in this stability regime. In the Suffield 2-92 m and 16-92 m layers all three laws appear to fit data equally well. The slight differences which do exist lack a systematic pattern and cannot be relied upon to determine the superiority of one law over another. This result appears to coincide with findings by Bernstein (1966) and Hansen (1966) when they analysed several sets of data in terms of different forms of the log-linear law.

The question of how significant are the differences between the power law, the logarithmic law, and the Monin-Obukhov log-linear law may be answered by studying Figs. 13-16. The vertical line segments represent the five per cent level confidence intervals for different laws. It may be noted from the diagrams that for Kerang data the Monin-Obukhov log-linear law is significantly better than the

power law in vicinity of $Ri = -0.03$ while in the other regions the confidence intervals overlap. For the Suffield data the significant differences occur in the 2-92 m layer between the Monin-Obukhov log-linear law and the logarithmic law in the interval $0.00 < Ri < 0.20$ and between the power law and the logarithmic law in vicinity of $Ri = -0.20$ and $Ri = 0.40$.

Figs. 17-20 show the confidence intervals for the Brooks log-power law, the modified log-linear law, and the Monin-Obukhov log-linear law. It appears that the confidence intervals for these three laws overlap for nearly all stability conditions. This result suggests that the differences among the two forms of the log-linear law and the log-power law are not significant at the five per cent level.

The relatively wide confidence intervals appearing in Figs. 17-20 may be somewhat misleading because, as shown in Appendix B, the width of the confidence intervals depends on the numbers of observations. This fact would suggest that the confidence intervals calculated not for each subgroup, but rather for each set of data may provide a better estimate of the significant differences between the different laws. Figs. 21-24 show the average weighted correlation coefficient, together with the corresponding confidence intervals, for each set of data. It may be observed from Fig. 21 that for the Kerang data the log-linear law is significantly better than either the logarithmic law or the power law. Whether the logarithmic law is significantly better than the power law is open to question because the five per cent level confidence intervals for these two laws just touch one another.

[illegible]

[illegible]

[illegible]

For the Suffield 0.5-16 m layer the power law is significantly better than the logarithmic law. However, there is no significant difference between these two laws and the log-linear law. In the Suffield 2-92 m layer the log-linear law is significantly better than the logarithmic law while there is no significant difference between the power law and either of the other two laws. Lastly, in the Suffield 16-92 m layer confidence limits from all three laws overlap, hence no one of the laws is significantly different from the other two.

The comparison among the average weighted correlation coefficients and the corresponding confidence intervals, for Brooks log-power law, modified log-linear law, and Monin-Obukhov log-linear law is indicated in Table 5.1. A study of Table 5.1 reveals that there are no significant differences among these three laws, a confirmation of the results shown in Figs. 17-20.

CHAPTER VII

CONCLUSIONS

The study of the wind profile measurements obtained at Suffield, Alberta, and at Kerang, Australia, shows that the value of the power law exponent p is smallest during slight and moderate superadiabatic conditions but increases slightly in extreme lapse conditions. Moreover, relatively large values of p exist during inversions. The numerical values of p obtained in this study are in agreement with those obtained at other locations by different investigators. It was found that the value of p remains approximately constant in the free convection regime and it increases rapidly in the forced convection regime.

The values of the parameters u_* and z_o appearing in the logarithmic and log-linear law, were evaluated from the Suffield wind profile data using the graphical method. The average values of u_* and z_o for Suffield terrain were estimated to be 0.34 m sec^{-1} and 3 cm respectively. In addition, the parameters u_*, z_o and D were calculated using Lettau's method. According to this method the average values of u_*, z_o and D, for Suffield terrain, are 0.34 m sec^{-1}, 1.5 cm and -3.2 cm respectively. Because of the nature of the vegetation cover at Suffield the value of z_o given by Lettau's method is considered more accurate than that given by the graphical method.

The parameters α, γ' and L_o, which appear in the Monin-Obukhov log-linear law, the Brooks log-power law, and the modified

log-linear law respectively, were calculated from the wind profile data. It was found that the value of the Monin-Obukhov constant α varies with Richardson number and also with the height of the layer considered. On the other hand, the parameter γ' was found to be independent of the height range but it varies with location and with stability. The length L_o was found to assume large values under unstable conditions and small values under inversions.

The comparisons among the power law, the logarithmic law, and the Monin-Obukhov log-linear law reveal that in the near-adiabatic conditions the log-linear law represents the observed data more accurately than either of the other two laws. However, when the conditions are not near-adiabatic the power law gives a better representation, although the difference is not significant at the five per cent level. The logarithmic law which is a special case of the log-linear law for $Ri = 0$, is found to be valid only under adiabatic conditions.

The comparisons among the Monin-Obukhov log-linear law, the Brooks log-power law, and the modified log-linear law show that the differences among these three laws are very slight and are not significant at the five per cent level. It would seem that the question, which of these three laws represents the data best, cannot be settled from the data which were available for this study.

Because of the tower interference in the Suffield 0.5-16 m layer wind measurements, discussed in Chapter VI, the results obtained for that layer are considered to be less significant than those

obtained for the other layers.

It may be observed from the graphs that the Kerang measurements display less scatter than do the Suffield data. This fact would suggest that a high degree of accuracy is required in the data to distinguish among the different wind profile models. Therefore, in the future experiments considerable care should be taken to ensure that a high degree of accuracy is achieved.

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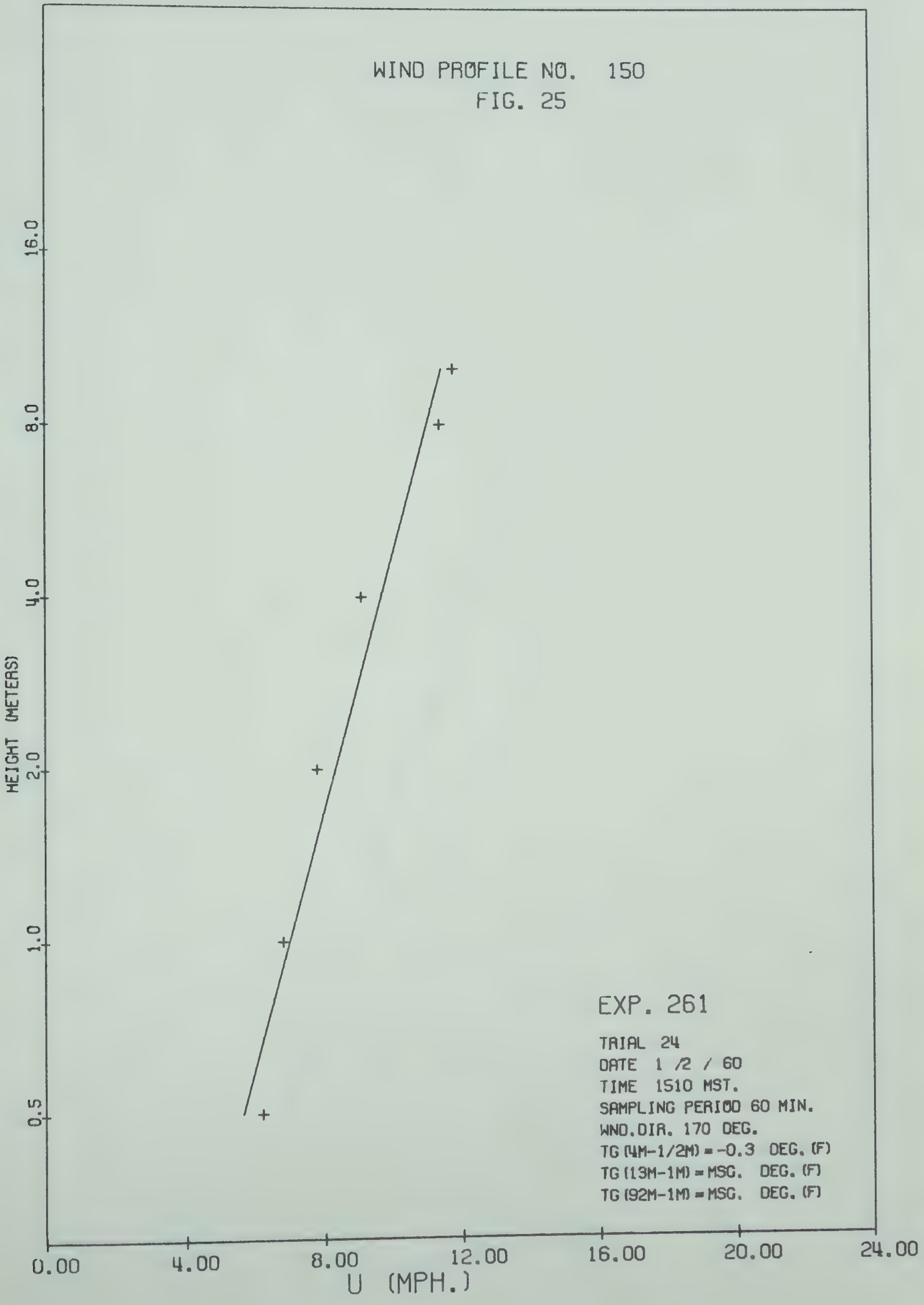
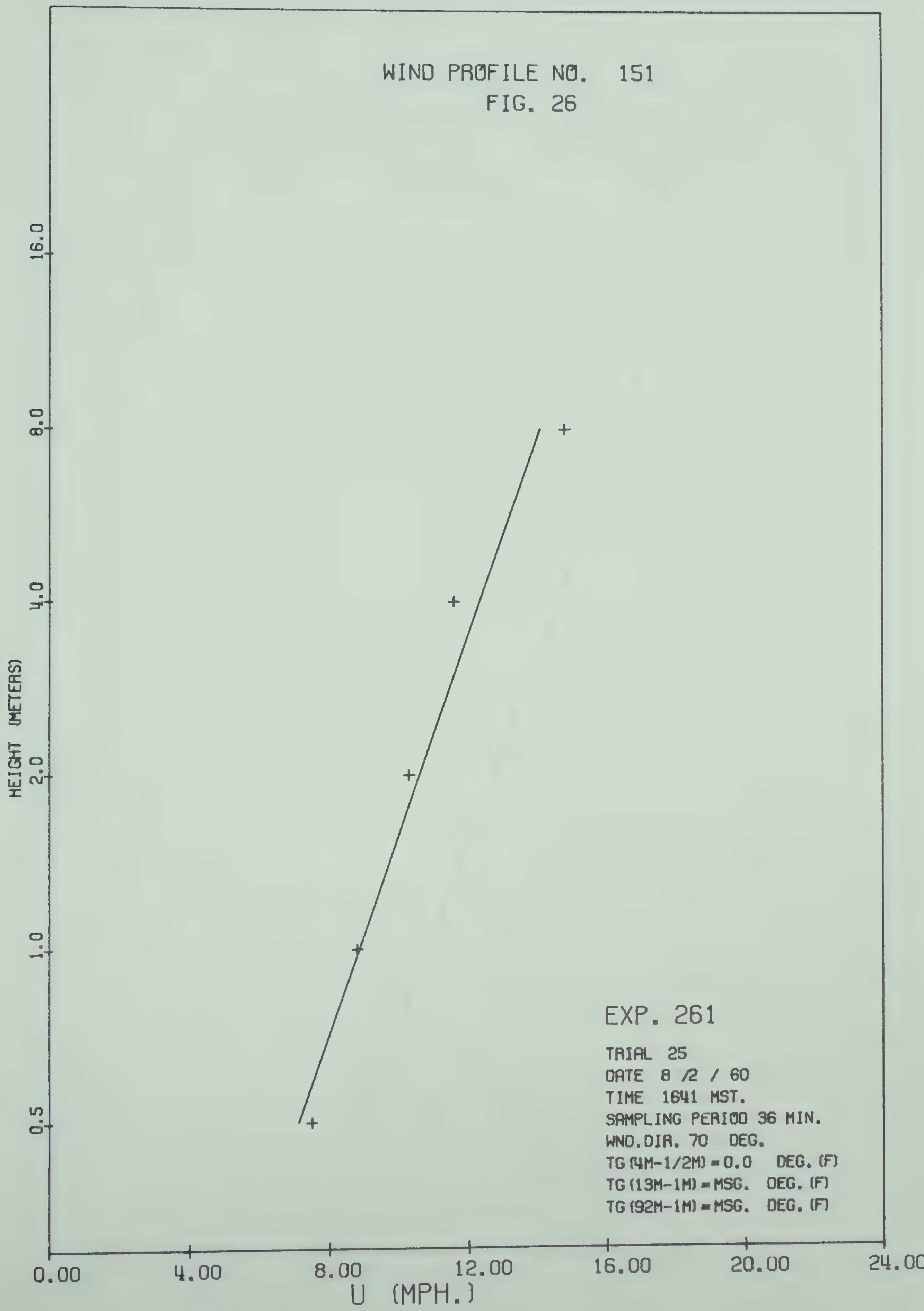
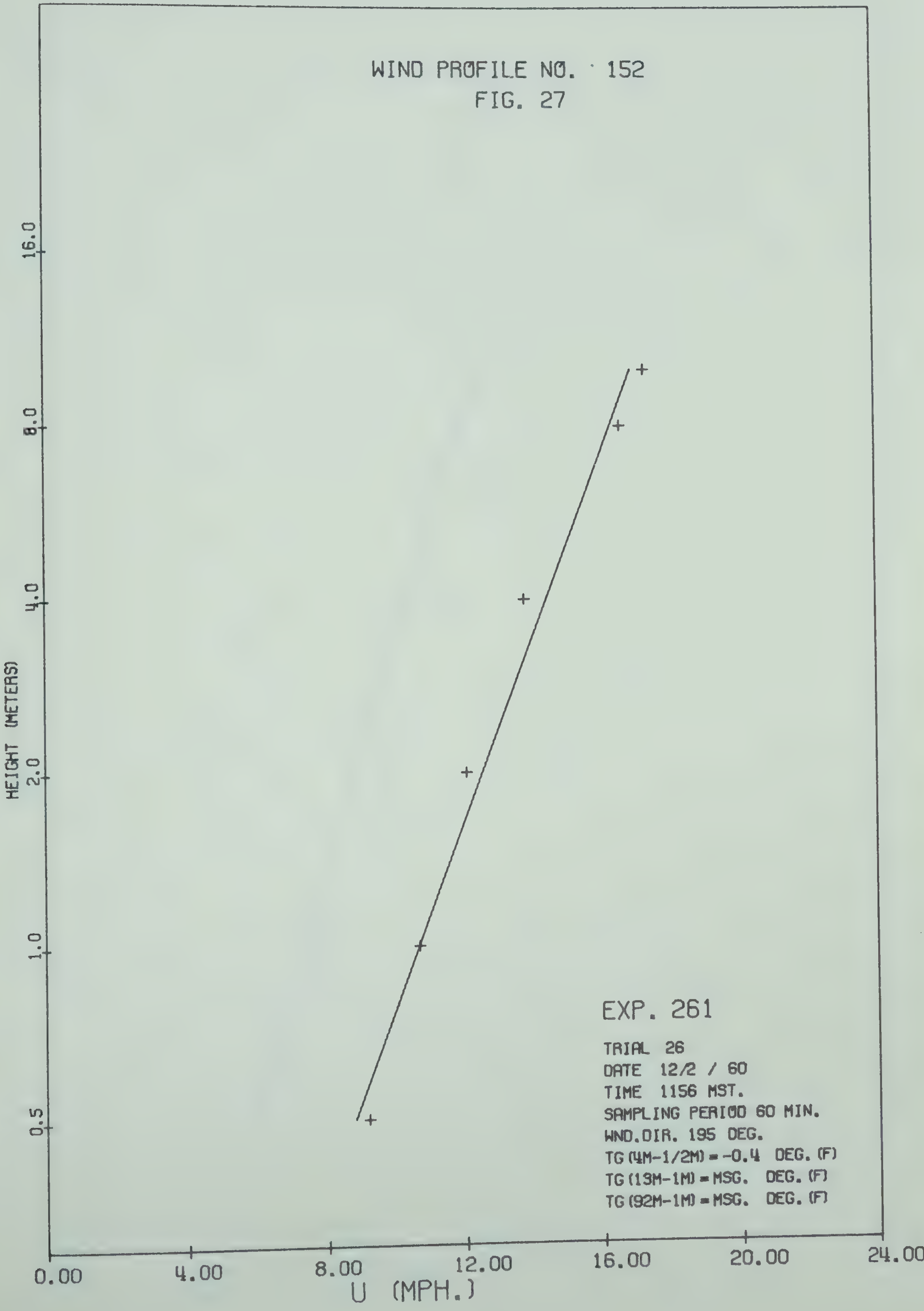
APPENDIX A

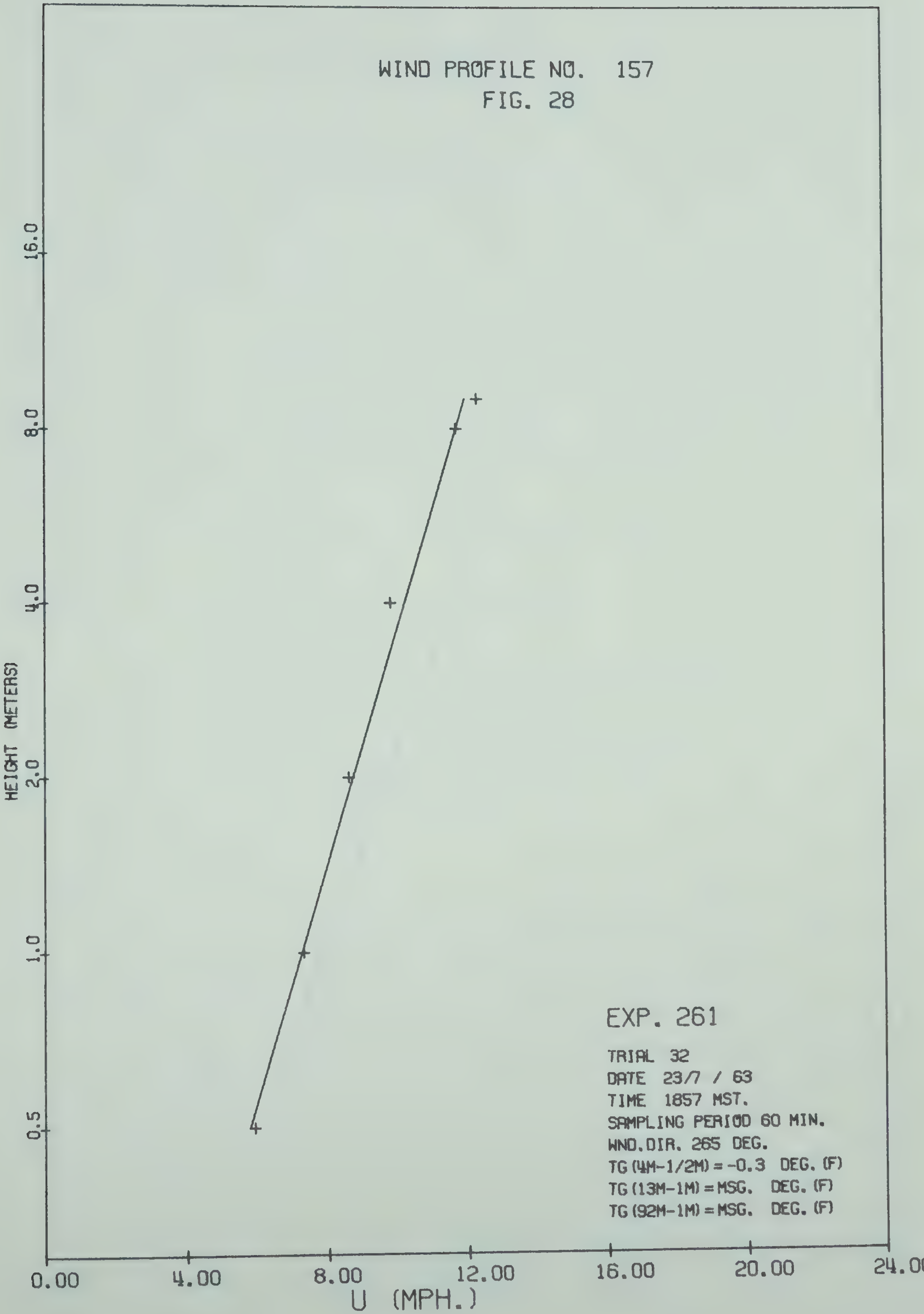
SUFFIELD WIND PROFILES FROM WHICH u_* AND z_o WERE OBTAINED

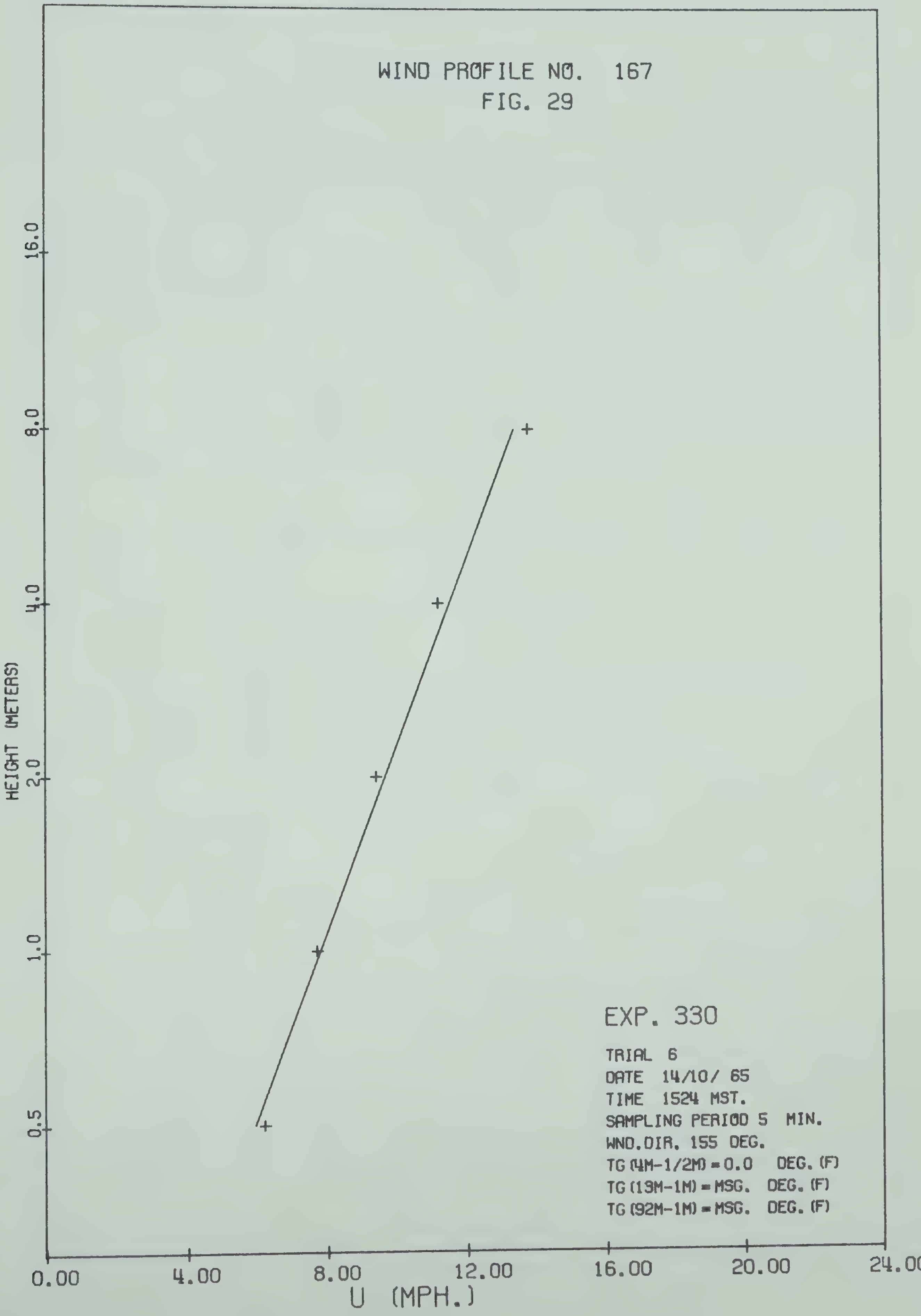

WIND PRØFILE NØ. 150
FIG. 25
HEIGHT (METERS)
16.0
8.0
4.0
2.0
1.0
0.5
0.00
4.00
8.00
12.00
16.00
20.00
24.00
U (MPH.)
EXP. 261
TRIAL 24
DATE 1 /2 / 60
TIME 1510 MST.
SAMPLING PERIØD 60 MIN.
WND.DIR. 170 DEG.
TG (4M-1/2M) = -0.3 DEG. (F)
TG (13M-1M) = MSG. DEG. (F)
TG (92M-1M) = MSG. DEG. (F)

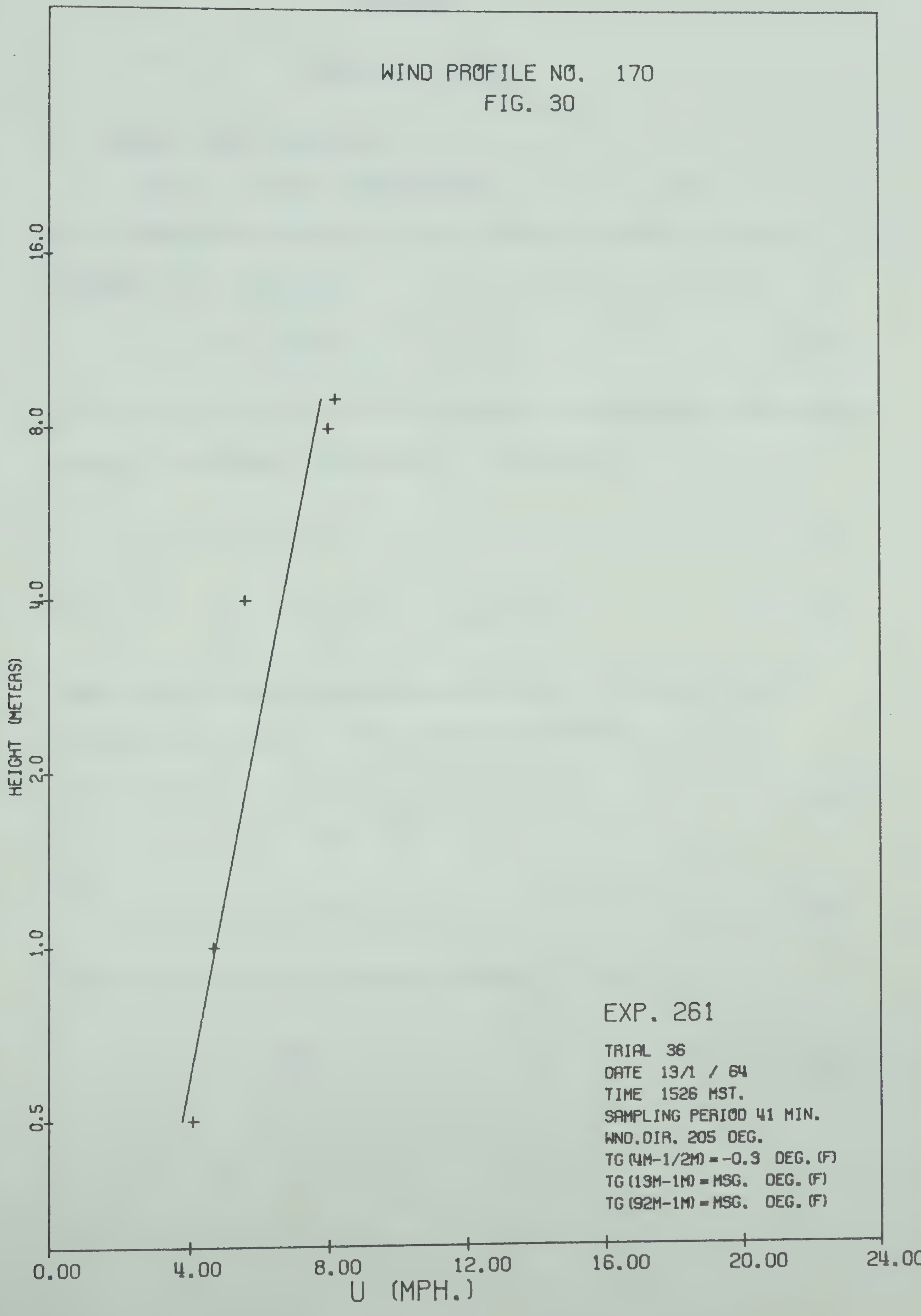


WIND PROFILE NO. 151
FIG. 26
HEIGHT (METERS)
16.0
8.0
4.0
2.0
1.0
0.5
EXP. 261
TRIAL 25
DATE 8 /2 / 60
TIME 1641 MST.
SAMPLING PERIOD 36 MIN.
WND.DIR. 70 DEG.
TG (4M-1/2M) = 0.0 DEG. (F)
TG (13M-1M) = MSG. DEG. (F)
TG (92M-1M) = MSG. DEG. (F)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
U (MPH.)


WIND PROFILE NO. 152
FIG. 27
HEIGHT (METERS)
16.0
8.0
4.0
2.0
1.0
0.5
0.00
4.00
8.00
12.00
16.00
20.00
24.00
U (MPH.)
EXP. 261
TRIAL 26
DATE 12/2 / 60
TIME 1156 MST.
SAMPLING PERIOD 60 MIN.
WND.DIR. 195 DEG.
TG (4M-1/2M) = -0.4 DEG. (F)
TG (13M-1M) = MSG. DEG. (F)
TG (92M-1M) = MSG. DEG. (F)


WIND PROFILE NO. 157
FIG. 28
HEIGHT (METERS)
16.0
8.0
4.0
2.0
1.0
0.5
0.00
4.00
8.00
12.00
16.00
20.00
24.00
U (MPH.)
EXP. 261
TRIAL 32
DATE 23/7 / 63
TIME 1857 MST.
SAMPLING PERIOD 60 MIN.
WND.DIR. 265 DEG.
TG (4M-1/2M) = -0.3 DEG. (F)
TG (13M-1M) = MSG. DEG. (F)
TG (92M-1M) = MSG. DEG. (F)


WIND PROFILE NO. 167
FIG. 29
HEIGHT (METERS)
16.0
8.0
4.0
2.0
1.0
0.5
0.00
4.00
8.00
12.00
16.00
20.00
24.00
U (MPH.)
EXP. 330
TRIAL 6
DATE 14/10/ 65
TIME 1524 MST.
SAMPLING PERIOD 5 MIN.
WND.DIR. 155 DEG.
TG (4M-1/2M) = 0.0 DEG. (F)
TG (13M-1M) = MSG. DEG. (F)
TG (92M-1M) = MSG. DEG. (F)


WIND PROFILE NO. 170
FIG. 30
HEIGHT (METERS)
16.0
8.0
4.0
2.0
1.0
0.5
EXP. 261
TRIAL 36
DATE 13/1 / 64
TIME 1526 MST.
SAMPLING PERIOD 41 MIN.
WND.DIR. 205 DEG.
TG (4M-1/2M) = -0.3 DEG. (F)
TG (13M-1M) = MSG. DEG. (F)
TG (92M-1M) = MSG. DEG. (F)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
U (MPH.)

APPENDIX B

STATISTICAL FORMULAE

1. Simple linear regression

Given n sets of observations (z_i, U_i), $i = 1, 2, \ldots n$, of an independent variable z and a dependent variable U then a straight line in the form:

$$U = az + b \tag{B.1}$$

can be fitted to these data by the method of least squares. The sample mean and the standard deviation of z are given by:

$$\bar{z} = \frac{1}{n} \Sigma z_i \tag{B.2}$$

and

$$s_z = \sqrt{\frac{1}{n-1} \Sigma (z_i - \bar{z})^2} \tag{B.3}$$

respectively. Similar expressions hold for U. The least-squares estimates of a and b in Eq. (B.1) are given as:

$$a = \frac{\Sigma (z_i - \bar{z}) \quad (U_i - \bar{U})}{\Sigma (z_i - \bar{z})^2} \tag{B.4}$$

and

$$b = U - a\bar{z} \tag{B.5}$$

The sample correlation coefficient between z and U is:

$$r = \frac{as_z}{s_U} \tag{B.6}$$

2. Fisher's z' - statistic

According to Brooks and Carruthers (1953), if the values of a correlation coefficient (r), computed from two or more rather short sets of data, are known a better value of r may be obtained by combining them. This is true only if it is reasonable to suppose that the different sets of data are samples from the same population. The correlation coefficients are combined by transforming them first into Fisher's z' - statistic given by:

$$z' = \frac{1}{2}\left[\ln (1 + r) - \ln (1 - r)\right] \tag{B.7}$$

This statistic has a nearly normal distribution and a standard error equal to:

$$\sigma = \frac{1}{(n - 3)^{\frac{1}{2}}} \tag{B.8}$$

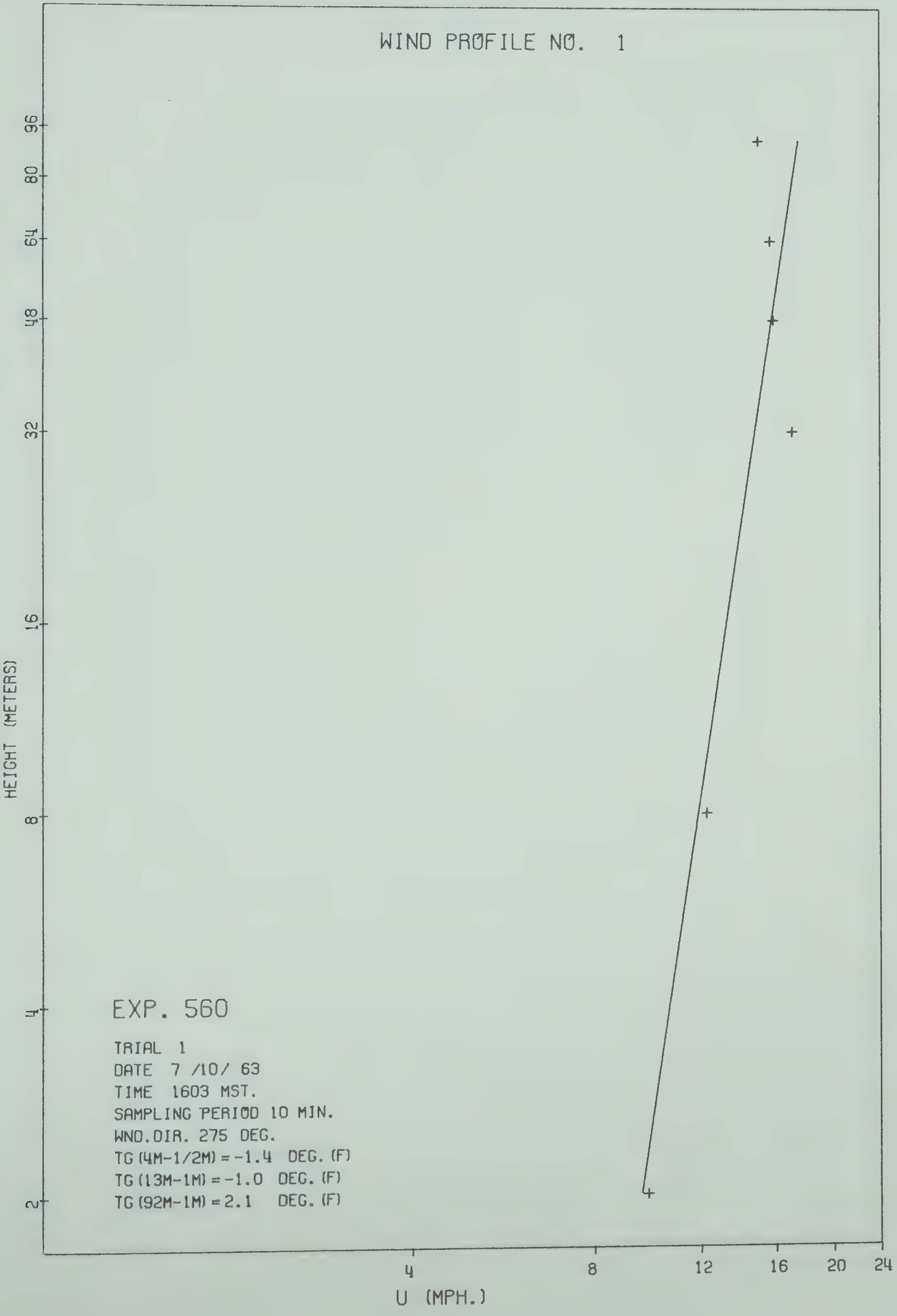
Next, a weighted mean of the z''s is found, the weights being proportional to $n - 3$. The weighted mean of z''s is converted back to r by equation:

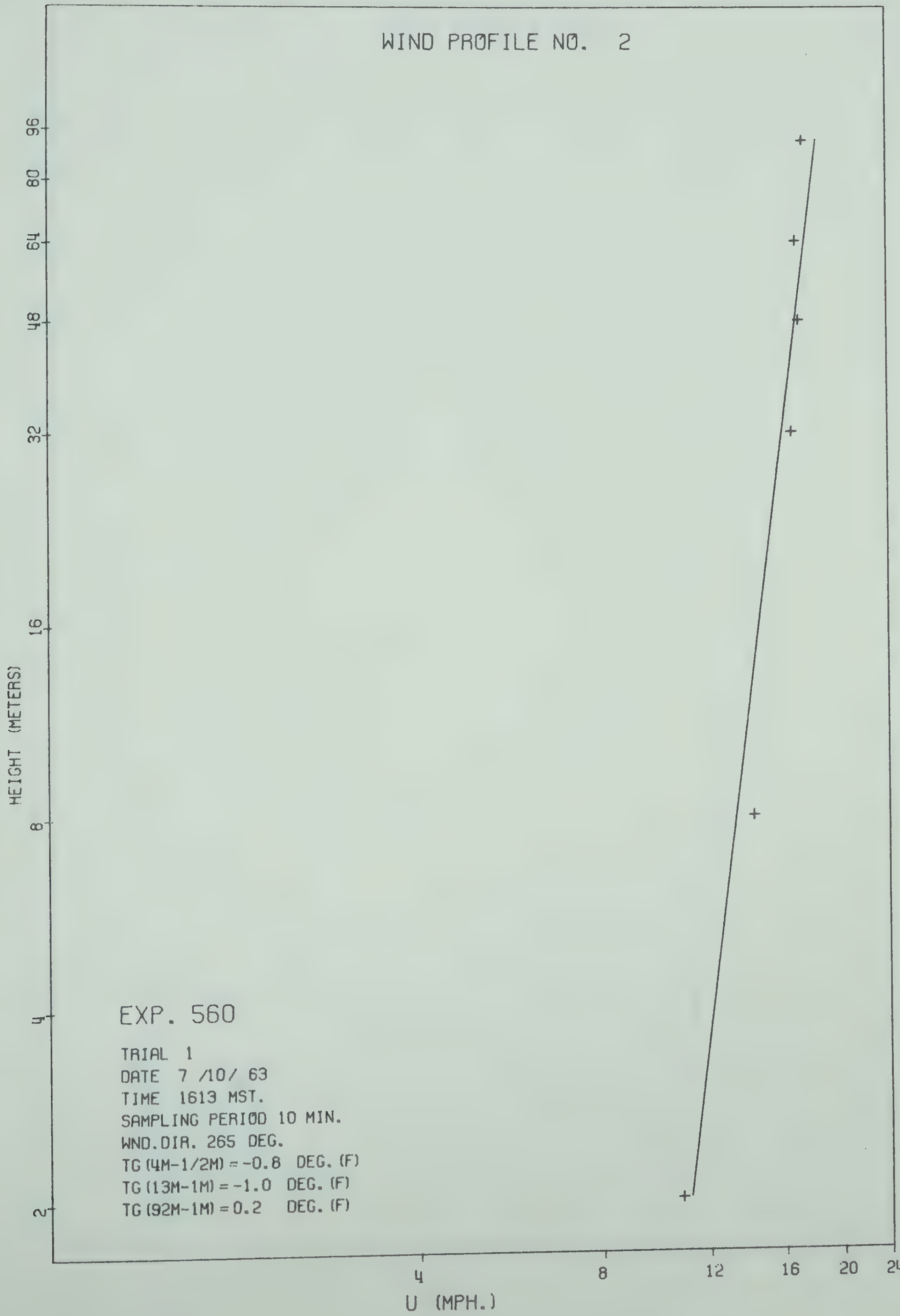
$$r = \frac{\exp (2z') - 1}{\exp (2z') + 1} \tag{B.9}$$

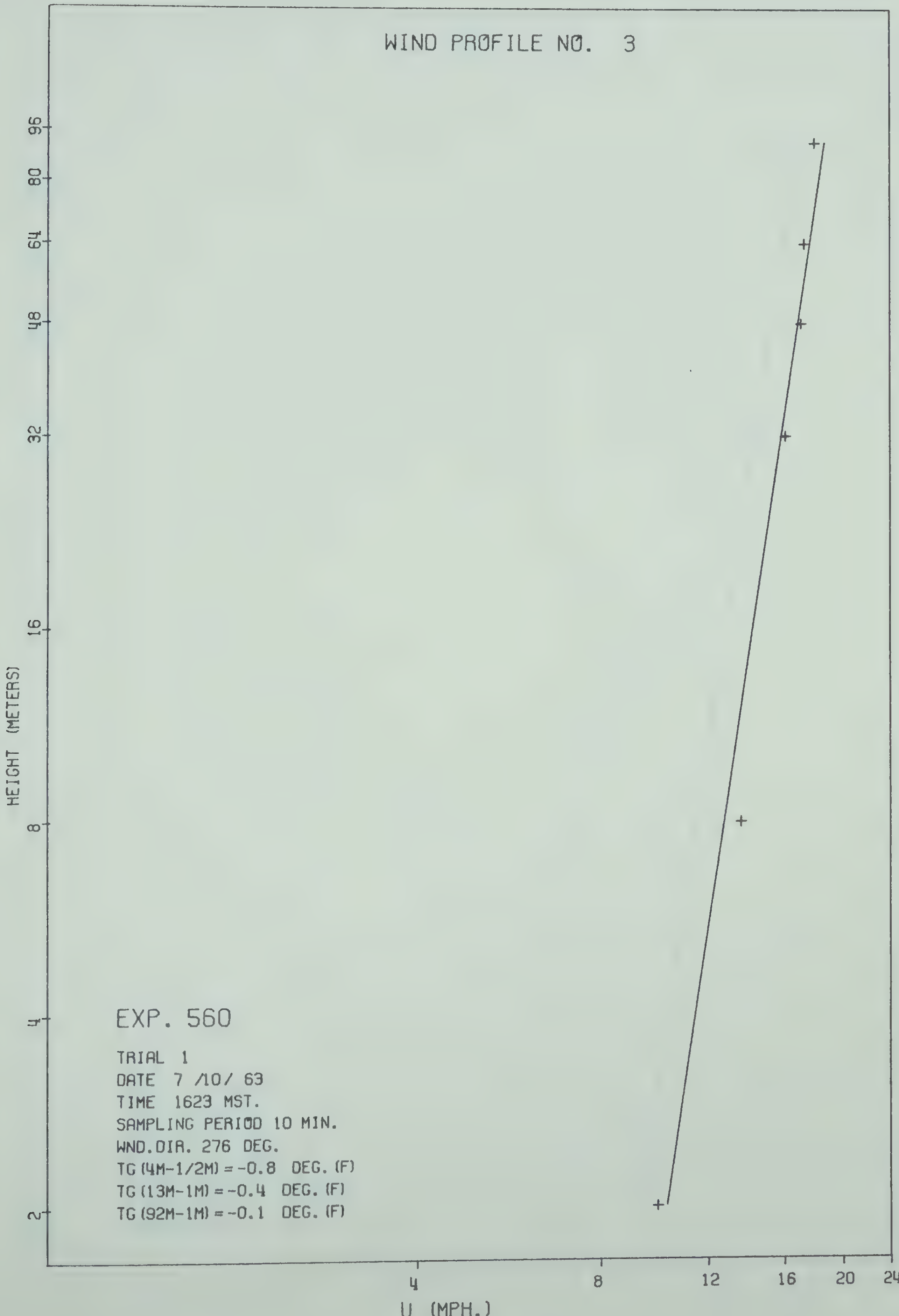
To calculate the 5% level confidence limits the values of $z' + 1.96\,\sigma$ and $z' - 1.96\,\sigma$ are calculated first and then transformed to r by Eq. (B.9). It may be noted that the relative size of confidence limits is dependent on the number of observations.

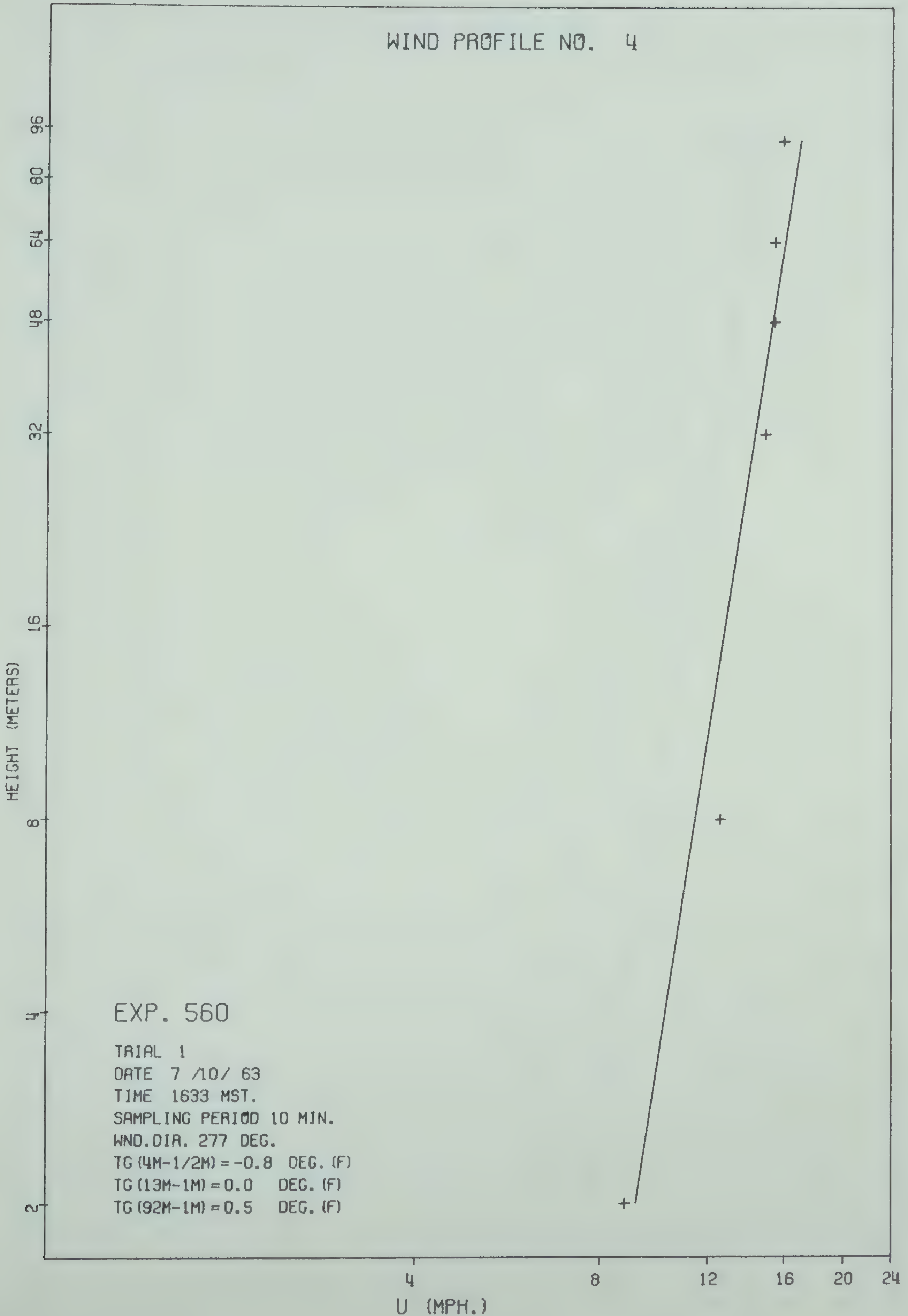
APPENDIX C

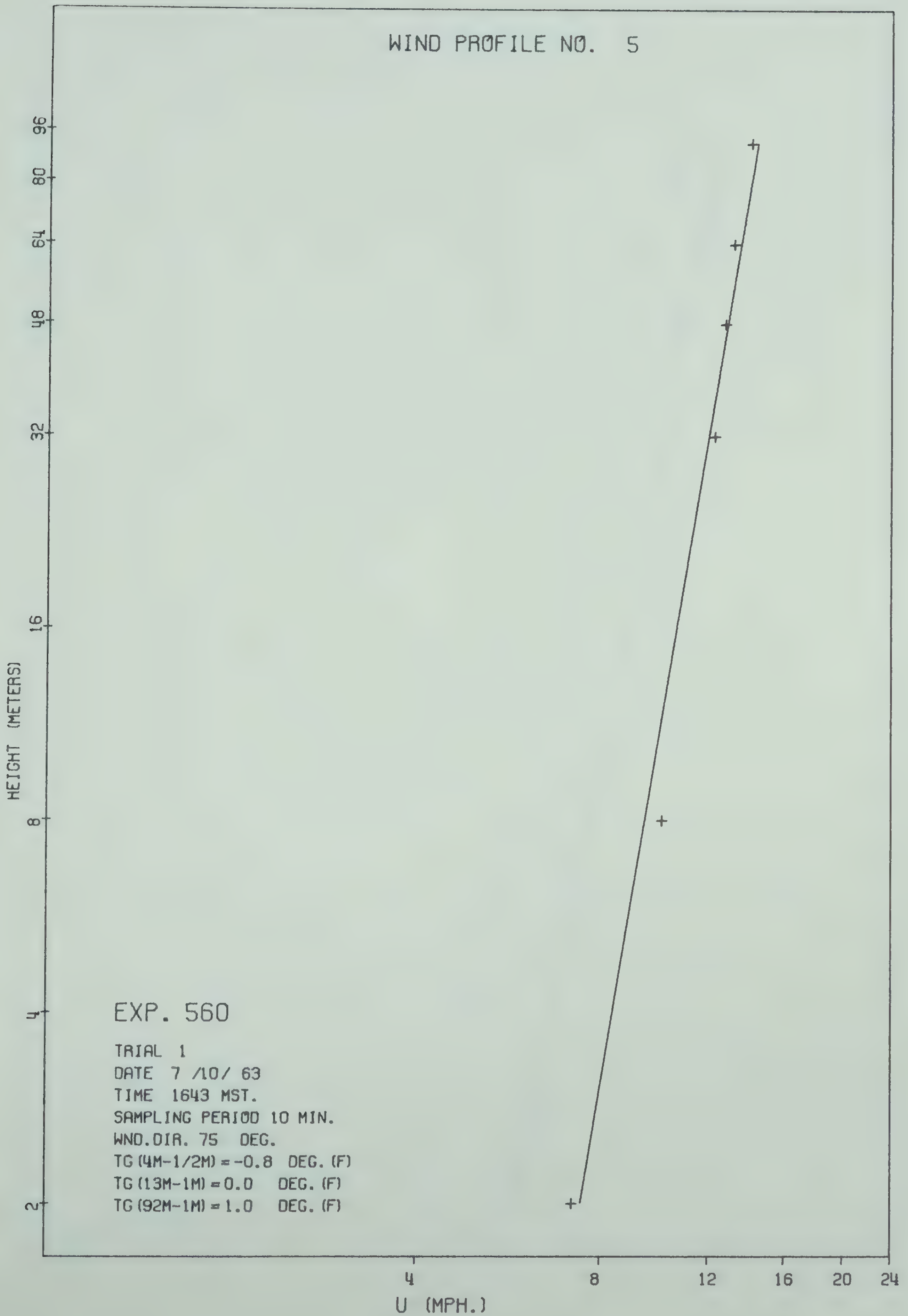
SELECTED WIND PROFILES FOR SUFFIELD 2 - 92 m LAYER

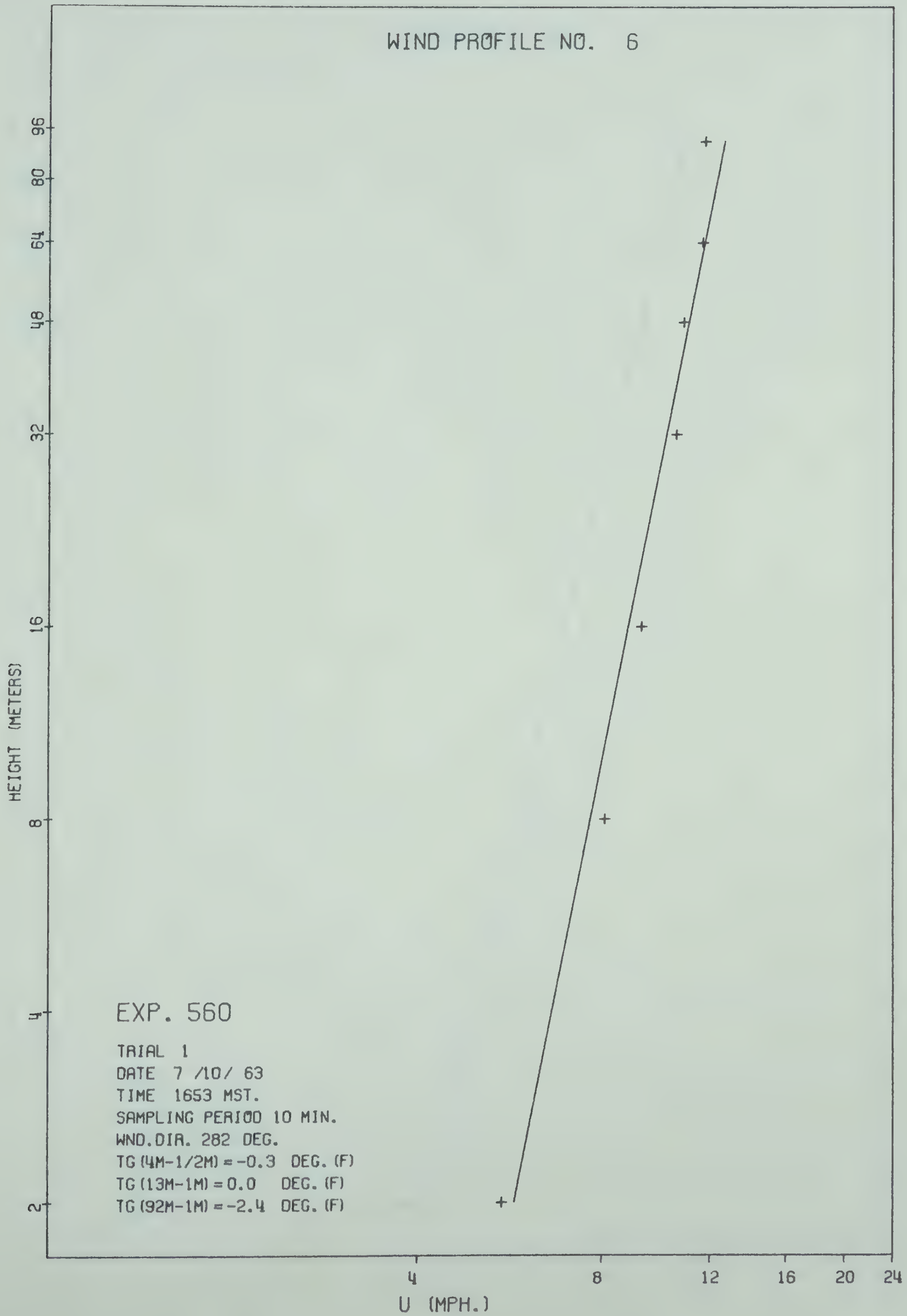

WIND PROFILE NO. 1
96
80
64
48
32
16
8
4
2
HEIGHT (METERS)
EXP. 560
TRIAL 1
DATE 7 /10/ 63
TIME 1603 MST.
SAMPLING PERIOD 10 MIN.
WND.DIR. 275 DEG.
TG (4M-1/2M) = -1.4 DEG. (F)
TG (13M-1M) = -1.0 DEG. (F)
TG (92M-1M) = 2.1 DEG. (F)
4
8
12
16
20
24
U (MPH.)

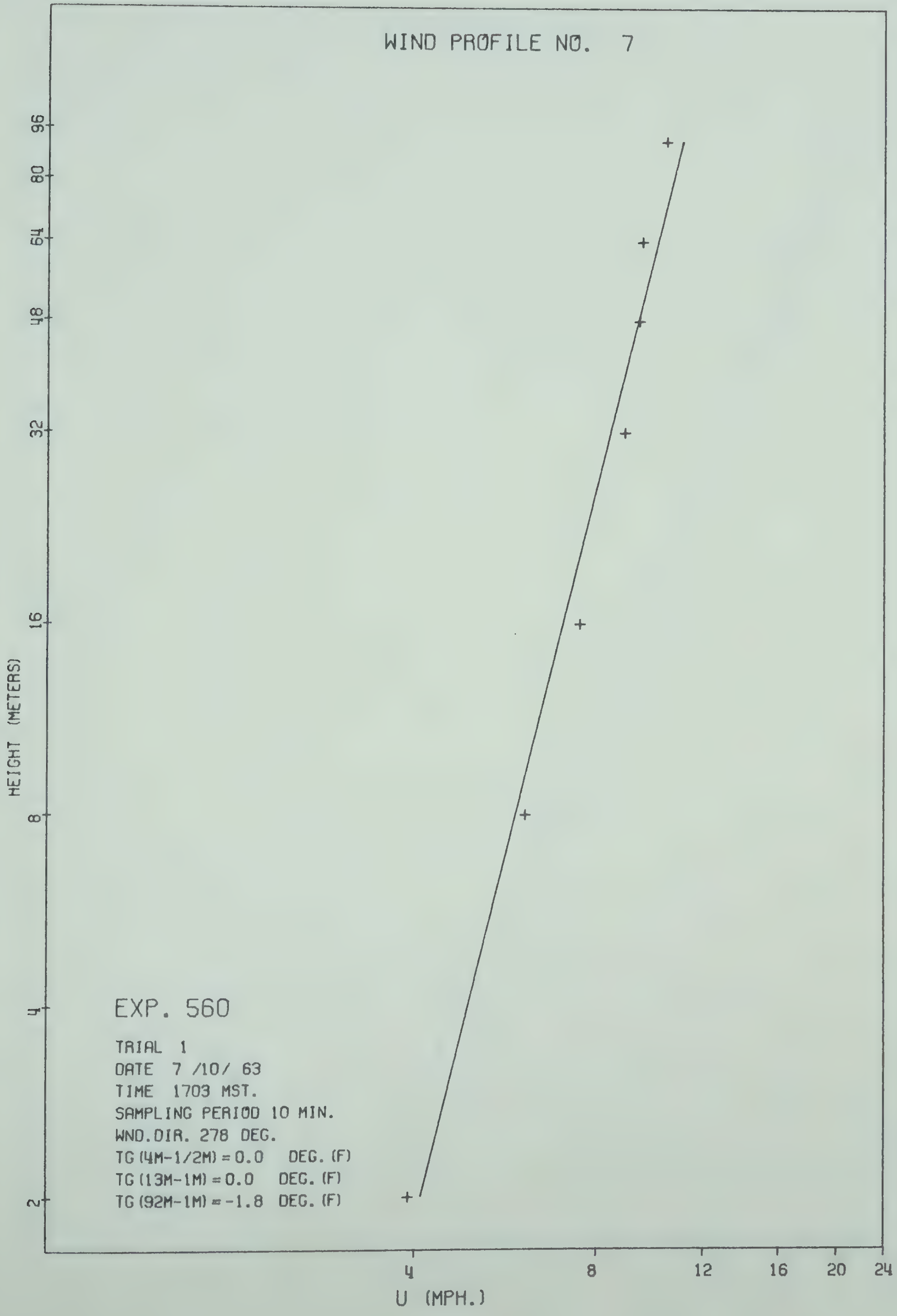

WIND PROFILE NO. 2
HEIGHT (METERS)
96
80
64
48
32
16
8
4
2
U (MPH.)
4
8
12
16
20
24
EXP. 560
TRIAL 1
DATE 7 /10/ 63
TIME 1613 MST.
SAMPLING PERIOD 10 MIN.
WND.DIR. 265 DEG.
TG (4M-1/2M) = -0.8 DEG. (F)
TG (13M-1M) = -1.0 DEG. (F)
TG (92M-1M) = 0.2 DEG. (F)

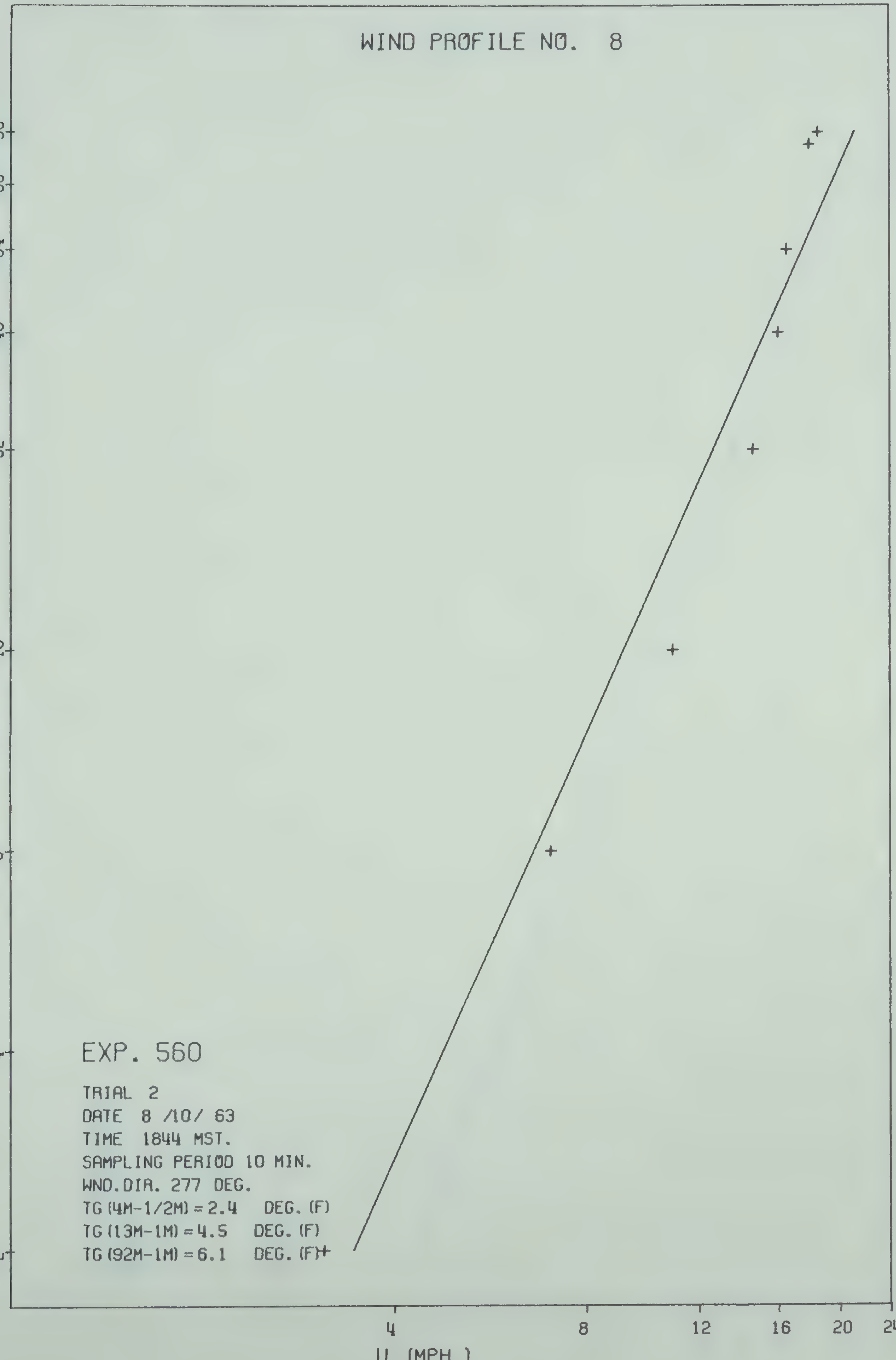

WIND PROFILE NO. 3
HEIGHT (METERS)
96
80
64
48
32
16
8
4
2
U (MPH.)
4
8
12
16
20
24
EXP. 560
TRIAL 1
DATE 7 /10/ 63
TIME 1623 MST.
SAMPLING PERIOD 10 MIN.
WND.DIR. 276 DEG.
TG (4M-1/2M) = -0.8 DEG. (F)
TG (13M-1M) = -0.4 DEG. (F)
TG (92M-1M) = -0.1 DEG. (F)

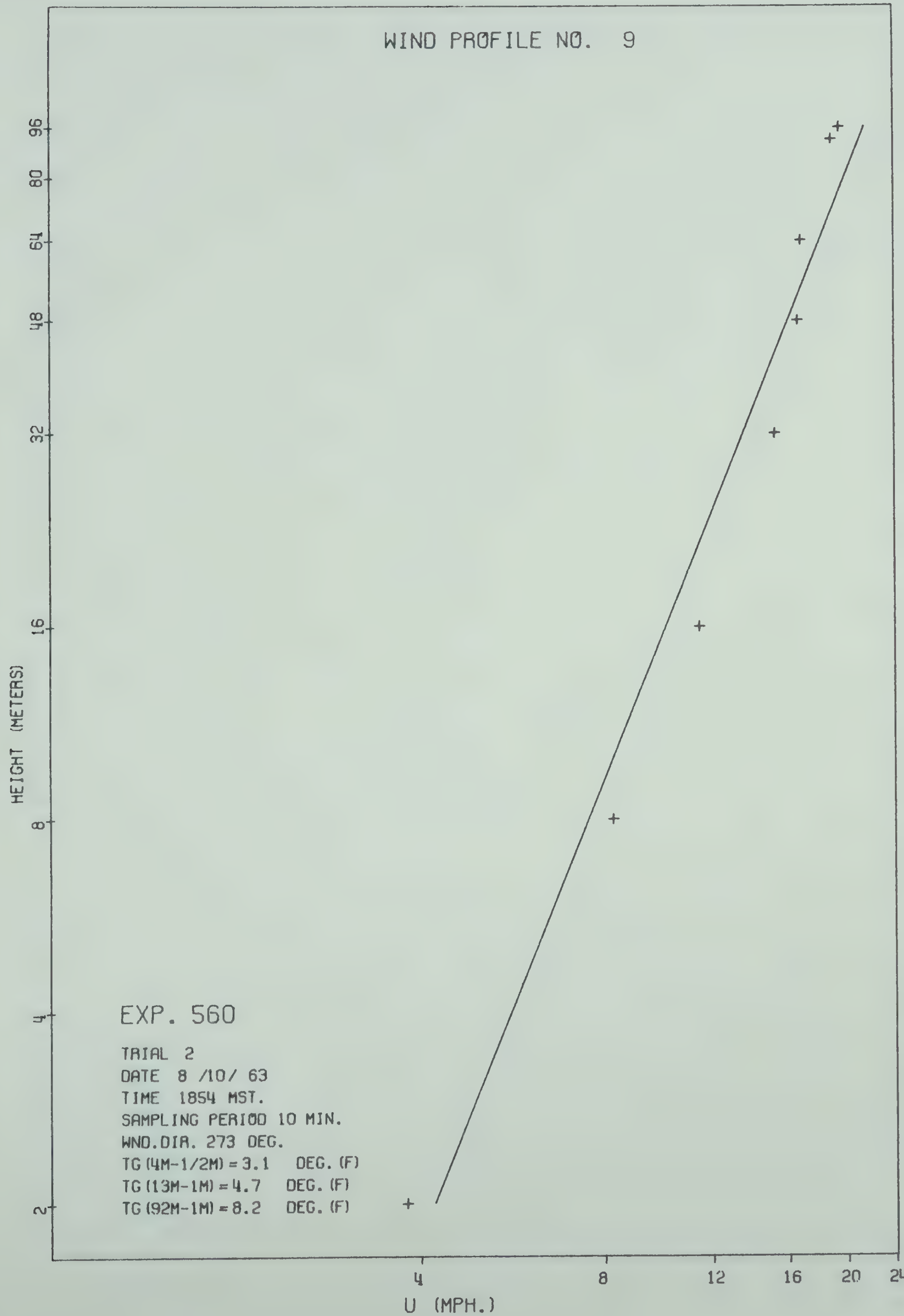

WIND PROFILE NO. 4
96
80
64
48
32
16
8
4
2
HEIGHT (METERS)
EXP. 560
TRIAL 1
DATE 7 /10/ 63
TIME 1633 MST.
SAMPLING PERIOD 10 MIN.
WND.DIR. 277 DEG.
TG (4M-1/2M) = -0.8 DEG. (F)
TG (13M-1M) = 0.0 DEG. (F)
TG (92M-1M) = 0.5 DEG. (F)
4
8
12
16
20
24
U (MPH.)

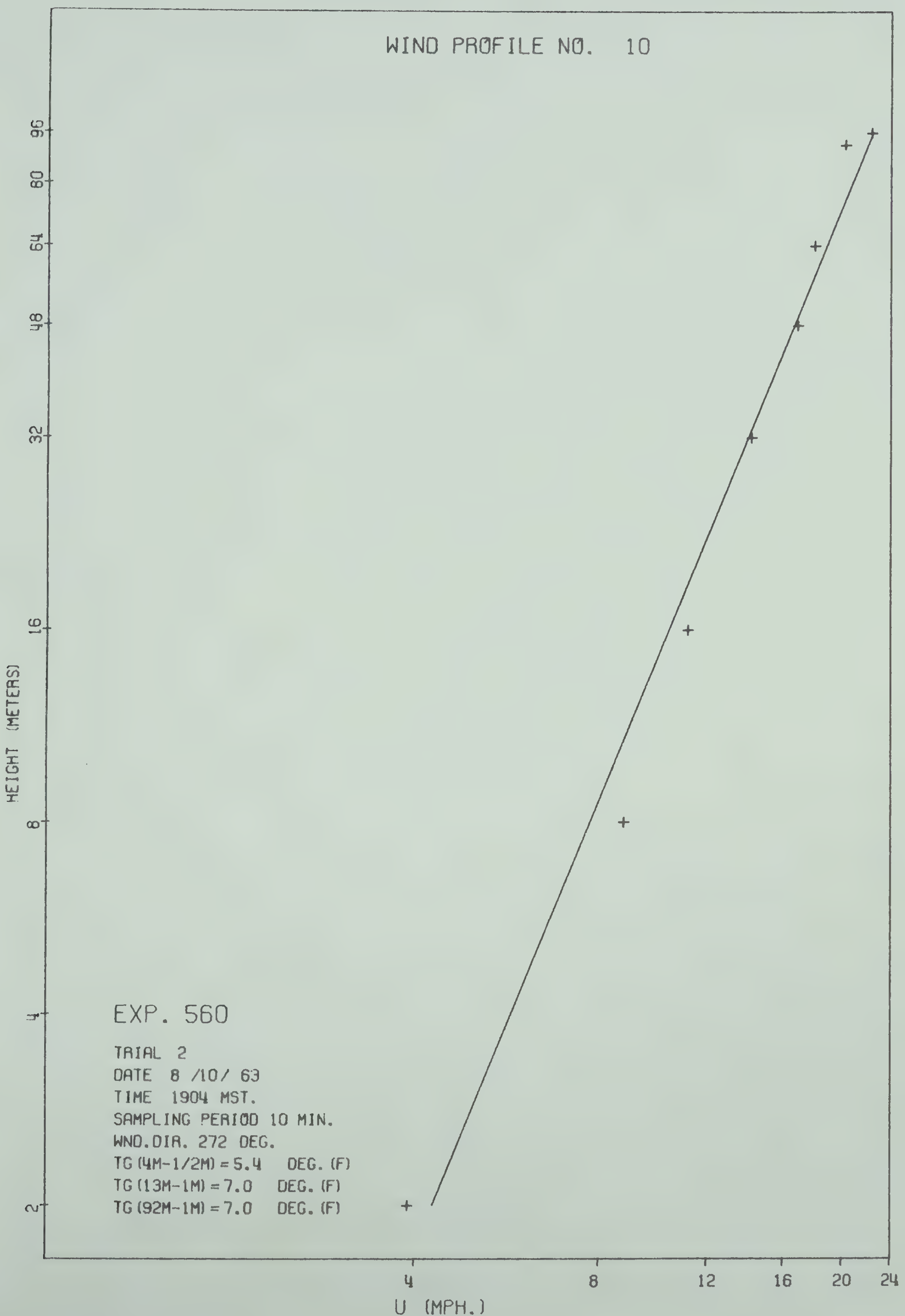

WIND PROFILE NO. 5
HEIGHT (METERS)
96
80
64
48
32
16
8
4
2
EXP. 560
TRIAL 1
DATE 7 /10/ 63
TIME 1643 MST.
SAMPLING PERIOD 10 MIN.
WND.DIR. 75 DEG.
TG (4M-1/2M) = -0.8 DEG. (F)
TG (13M-1M) = 0.0 DEG. (F)
TG (92M-1M) = 1.0 DEG. (F)
4
8
12
16
20
24
U (MPH.)


WIND PROFILE NO. 6
96
80
64
48
32
16
8
4
2
HEIGHT (METERS)
4
8
12
16
20
24
U (MPH.)
EXP. 560
TRIAL 1
DATE 7 /10/ 63
TIME 1653 MST.
SAMPLING PERIOD 10 MIN.
WND.DIR. 282 DEG.
TG (4M-1/2M) = -0.3 DEG. (F)
TG (13M-1M) = 0.0 DEG. (F)
TG (92M-1M) = -2.4 DEG. (F)


WIND PROFILE NO. 7
HEIGHT (METERS)
96
80
64
48
32
16
8
4
2
U (MPH.)
4
8
12
16
20
24
EXP. 560
TRIAL 1
DATE 7 /10/ 63
TIME 1703 MST.
SAMPLING PERIOD 10 MIN.
WND.DIR. 278 DEG.
TG (4M-1/2M) = 0.0 DEG. (F)
TG (13M-1M) = 0.0 DEG. (F)
TG (92M-1M) = -1.8 DEG. (F)


WIND PROFILE NO. 8
96
80
64
48
32
16
8
4
2
HEIGHT (METERS)
EXP. 560
TRIAL 2
DATE 8 /10/ 63
TIME 1844 MST.
SAMPLING PERIOD 10 MIN.
WND.DIR. 277 DEG.
TG (4M-1/2M) = 2.4 DEG. (F)
TG (13M-1M) = 4.5 DEG. (F)
TG (92M-1M) = 6.1 DEG. (F)
4
8
12
16
20
24
U (MPH.)


WIND PROFILE NO. 9
HEIGHT (METERS)
96
80
64
48
32
16
8
4
2
EXP. 560
TRIAL 2
DATE 8 /10/ 63
TIME 1854 MST.
SAMPLING PERIOD 10 MIN.
WND.DIR. 273 DEG.
TG (4M-1/2M) = 3.1 DEG. (F)
TG (13M-1M) = 4.7 DEG. (F)
TG (92M-1M) = 8.2 DEG. (F)
4
8
12
16
20
24
U (MPH.)


WIND PROFILE NO. 10
HEIGHT (METERS)
96
80
64
48
32
16
8
4
2
U (MPH.)
4
8
12
16
20
24
EXP. 560
TRIAL 2
DATE 8 /10/ 63
TIME 1904 MST.
SAMPLING PERIOD 10 MIN.
WND.DIR. 272 DEG.
TG (4M-1/2M) = 5.4 DEG. (F)
TG (13M-1M) = 7.0 DEG. (F)
TG (92M-1M) = 7.0 DEG. (F)